MTH 3110: Abstract Algebra 1 - Fall Semester 2018
Problem List 1

Prof: Mandi Schaeffer Fry

Due 9/3/18

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted without the use of a "full redo". Solutions should be submitted at **the beginning of class** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!) Multiple pages should be stapled. (I will NOT provide the stapler.) Points will be deducted from assignments not following these guidelines. Recall that if you use LaTeX (including Overleaf) to typeset your homework, you'll get an extra "free redo".

1 Before Class On...

- Mon, 9/3: Read Chapter 2
- Need to review some things from Intro to Proofs? Flip back through your text from that course or look into *Book of Proof* by Richard Hammack. It's a free textbook found at `https://www.people.vcu.edu/~rhammack/BookOfProof/` and is reasonably close to the book you likely used in MTH 3100. Things to refresh:
  - direct proof;
  - proof by contradiction and negating statements;
  - contrapositive statements;
  - proving existence and uniqueness;
  - sets: subsets, intersections, unions, proving containment or equality.

2 Notation and Definitions to Know

An $*$ denotes Flashquiz - eligible items.

- the general notation surrounding functions:

  e.g. $f\colon A \to B$, $a \mapsto b$
- domain, codomain, *image of a function
- *surjective function
- *injective function
- *bijective function

3 For Practice...

- Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. If possible, write down examples of functions $f\colon A \to B$ that are (a) injective but not surjective, (b) surjective but not injective, (c) neither surjective nor injective, (d) bijective. If some of these are not possible, explain why.
- Answer the same questions, but with $f\colon B \to A$.
- Let A and B be two sets. Let $f\colon A \to B$ and $g\colon B \to C$ be functions. Prove that if $g \circ f$ is bijective, then g is surjective and f is injective.
- Chapter 1, exercises 1-4 (from Gallian **6th** edition - compare with me or a classmate to make sure you have the right ones.)

4 To Turn In On 9/3/18 at the Beginning of Class

1. Let $f\colon A \to B$ and $g\colon B \to C$ be functions. Prove that
   (a) If f is surjective and g is surjective, then $g \circ f$ is surjective.
   (b) If f is bijective and g is bijective, then $g \circ f$ is bijective. (Recall that we proved in class that f injective and g injective $\Rightarrow g \circ f$ injective. You may use this fact.)
2. (a) Give an example of two elements $f, g \in S_3$ such that $f \circ g \neq g \circ f$ (and prove that your example works).
   (b) Let $n \geq 3$. Use part (a) to prove that $\circ$ is not commutative on S_n. (That is, show that there exist $f, g \in S_n$ such that $f \circ g \neq g \circ f$.)

Recall the following definitions for problem 2:

Definition. Let A be a nonempty set. Then a **permutation of A** is a bijection from A to A. The **symmetric group on n symbols** is the set of all permutations on the set $\{1, ..., n\}$. That is,

$$S_n := \{f\colon \{1, ..., n\} \to \{1, ..., n\} \mid f \text{ is a bijection}\}$$