DR. MANDI'S LECTURE OUTLINE
SECTION 5.3 REVIEW

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Date:

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Example 1. Let $f(x) = 6x$. Let's think about the following problems for $f(x)$. What are their similarities and differences?

(1) Determine an antiderivative of $6x$.

(2) Determine $\int 6x dx$.

For the next ones, use
(a) a picture and your knowledge of geometry; and
(b) calculus.

(3) Determine the area between the graph of $f(x) = 6x$ and the x-axis on the interval $[0, 5]$.

(4) Determine the area between the graph of $f(x) = 6x$ and the x-axis on the interval $[-5, 5]$.

(5) Determine the "net area" between the graph of $f(x) = 6x$ and the x-axis on the interval $[-5, 5]$.

Think about it!. How did our computations compare?

Think about it!. Do you remember the name of the theorem that lets us calculate definite integrals?

THE FUNDAMENTAL THEOREM OF CALCULUS

Theorem (The Fundamental Theorem of Calculus, "Version 2"). *Let f be continuous on $[a, b]$.*

If F is ____ *then*

$$\int_a^b f(x)dx =$$

In other words,

$$\int_a^b F'(x)dx =$$

Notation. We often write

$$[F(x)]_a^b \quad \text{or more simply} \quad F(x)\Big|_a^b$$

for the expression $F(b) - F(a)$.

Think about it!. Does the choice of F matter?

Think about it!. What is $\int_a^a f(x)dx$?

Example 2. Evaluate $\int_0^1 (x^2 + \sqrt{x})dx$.

Example 3. Find the area between $y = x^2 - 4$ and the x-axis from $x = 2$ to $x = 4$.

Example 4. Find

(a) the "net area" and
(b) the area

between the x-axis and $x^2 - 4$ on $[0, 4]$.

Think about it!. In Example 1, how would you express the area between $f(t) = 6t$ and the x axis on the interval $[0, x]$?

Definition 1. Given an integrable function f, the ______________________________ is

$$A(x) =$$

Think about it!. What is $A'(x)$?

Theorem (The Fundamental Theorem of Calculus, "Version 1"). *Let f be continuous on $[a, b]$. Then $A(x) = \int_a^x f(t)dt$ is ____________________ on $[a, b]$ and ____________________ on (a, b) and*

$$A'(x) =$$

***Fun Fact!**.* In other words, the area function $A(x)$ is an ________________ of f!!!

Think about it!. Why doesn't the starting point a matter?

Example 5. Find $\frac{dy}{dx}$, where $y = \displaystyle\int_0^x (t^4 - 8t + 2)dt$.

Example 6. Find $\frac{dy}{dx}$, where $y = \displaystyle\int_x^\pi \cos(t^2)dt$.

Example 7. Find $\frac{dy}{dx}$, where $y = \displaystyle\int_2^{x^2} \sin(t)dt$.