MTH 3110: Abstract Algebra I
Fall Semester 2018
Problem List 3

Prof: Mandi Schaeffer Fry

Due 9/21/18

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted. Solutions should be submitted to me in my office or via email **by 4pm on Friday, Sept. 21** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!) Multiple pages should be stapled. (I will NOT provide the stapler.) Points will be deducted from assignments not following these guidelines. You also have the option to LaTeX your homework for extra credit.

Special rules just for HW 3:

We agreed in class to change the **exam date to Wednesday, September 26**. It would be useful to be able to talk about solutions to homework (including HW3) on Monday, Sept 24 for review. But then I can't let you do a redo! So

- If you want to be able to do a redo for HW3, hand it in during class on **Wednesday, 9/19** (or email it by 6pm that day).
- If you don't turn it in by 9/19, you may *not* do a redo on HW3. I will hand back the graded assignment on Monday 9/24 before we review.
- You may either come by on Friday 9/21 to get your graded HW 3 back OR email me (before Friday at 3pm) to let me know you can't come by, and I'll scan and email your graded assignment.
- Redos will be due on Monday, 9/24 in class.
- Everyone who turns HW3 in by Wednesday 9/19 at 6pm may do a full redo for free (I won't count it against your redo count).

1 Notation and Definitions to Know

An $*$ denotes Flashquiz - eligible items.

- *subgroup
- *cyclic group
- *be able to state the "1-step test"
- *be able to state the "2-step test"

2 Before Class On...

- Wed, 9/12: Read Chapter 3
- Mon, 9/17: Read Chapter 4

3 For Practice...

All exercises listed as practice problems are from Gallian **6th** edition - compare with me or a classmate to make sure you have the right ones.

- Chapter 3, exercises 1, 4-8, 13, 15, 20, 28, 34

4 To Turn In by 6pm on 9/19/18 for the option of a redo, or by 4pm on 9/21/18 for no option of a redo

1. In each of the following, determine the order of the group and the order of each element in the group. (You may just write them down, this does not have to be in proof format.)
   (a) $\mathbb{Z}_{12}$
   (b) $U(6)$
   (c) D_4 (the symmetries of a square - you can refer to the Cayley table we did in class)
   (d) What relationship do you see between the order of an element of a group and the order of the group?
2. For each divisor $k > 1$ of $n \in \mathbb{N}$, let $U_k(n) := \{x \in U(n) | x \mod k = 1\}$.
   (a) List the elements of $U_4(12)$ and $U_5(20)$. (This also does not have to be in proof format.)
   (b) Show that $U_k(n)$ is a subgroup of $U(n)$.
3. Let G be a group and let H and K be subgroups of G. Show that $H \cap K$ is a subgroup of G. (Recall that $H \cap K$ is the intersection: $\{x | x \in H \text{ and } x \in K\}$.