MTH 3110: Abstract Algebra I
Fall Semester 2018
Problem List 5

Prof: Mandi Schaeffer Fry

Due 10/26/18

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted. Solutions should be submitted to me in my office or by email **by 4pm on Friday, Oct 26** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!) Multiple pages should be stapled. (I will NOT provide the stapler.) Points will be deducted from assignments not following these guidelines. You also have the option to LaTeX your homework for extra credit.

1 Before Class On...

- Mon, 10/22: Read Chapter 5 through Theorem 5.3.

2 Notation and Definitions to Know

An $*$ denotes Flashquiz - eligible items for a Flashquiz Wednesday, 10/24/18.

- symmetric group S_n
- cycle notation
- *disjoint cycle
- *cyclic group
- *generator

3 For Practice...

All exercises listed as practice problems are from Gallian **6th** edition - compare with me or a classmate to make sure you have the right ones.

- Chapter 4, exercises 1-5, 10, 14, 25, 44, 48, 54
- Chapter 5, exercises 1-5, 8, 17, 18, 28, 36,
- Let $\langle a\rangle, \langle b\rangle$, and $\langle c\rangle$ be cyclic groups of orders $6, 8$, and 20, respectively. Find all generators of $\langle a\rangle, \langle b\rangle$, and $\langle c\rangle$. (Prove your answers)
- Let $n > 1$ and let $\alpha \in S_n$ be an m-cycle. What is α^{-1}?
- Represent the symmetries of an equilateral triangle (that is, D_3) as a group of permutations of its vertices. Do you notice anything interesting?

(Turn-In problems on back)

4 To Turn In by 4pm on 10/26/18

1. Determine the subgroup lattice for $\mathbb{Z}_{18}$.
2. Let $|x| = 40$. List all elements of $\langle x \rangle$ that have order 10. Show how you got your answer - make sure to note any theorems you are using.
3. Prove that if G is a group of order 3, then G is cyclic. (Hint: Let $G = \{1, a, b\}$. What are the options for $ab \in G$?)
4. Let G be a group and $a, b \in G$. Show that if $|a|$ and $|b|$ are relatively prime, then $\langle a \rangle \cap \langle b \rangle = \{e\}$. (Hint: remember on HW 2 you showed that if $H, K \leq G$, then $H \cap K \leq G$. Can you extend this to showing $H \cap K \leq H$? What do you know about the order of subgroups of cyclic groups?)
5. Let $n > 1$ and let $\alpha \in S_n$ be an m-cycle. Prove that $|\alpha| = m$.

(Bonus question!) Let $n \geq 3$. Show that $Z(S_n) = \{(1)\}$, where (1) denotes the identity permutation.