MTH 3110: Abstract Algebra I
Fall Semester 2018
Problem List 6

Prof: Mandi Schaeffer Fry

Due 11/28/18

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted. Solutions should be submitted by email **on Wednesday, Nov 28** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!)

1 Before Class On...

- Wed, 11/14: Read Chapter 5
- Mon, 11/26: Read Chapter 7

2 Notation and Definitions to Know

An $*$ denotes Flashquiz - eligible items for a Flashquiz Monday, 11/26/18.

- symmetric group S_n
- *even permutation, *odd permutation
- *alternating group A_n
- *statement of Cayley's Theorem
- *coset

3 For Practice...

All exercises listed as practice problems are from Gallian **6th** edition - compare with me or a classmate to make sure you have the right ones.

- Chapter 5, exercises 9, 14, 18-21, 28, 36, 46, 50
- Chapter 7, exercises 1-5, 14-16
- Determine the cycle forms of the elements of $D_3, U(10)$, and $U(20)$ in their left regular representations.
- Let G be a group and H a subgroup. Prove that $g_1H = g_2H \iff g_1^{-1}g_2 \in H$. Recall that the left coset of G corresponding to $g \in G$ with respect to H is

$$gH := \{gh \mid h \in H\}$$

(Turn-In problems on back)

4 To Turn In by email on 11/28/18

1. Let $n \geq 2$, and let $\alpha \in S_n$ have order m. Show that if m is odd, then $\alpha \in A_n$.
2. Prove $\Phi : S_n \to \{\pm 1\}$, given by $\Phi(g) = 1$ if g is even and $\Phi(g) = -1$ if g is odd, is a homomorphism. What is $\ker \Phi$?
3. Let $\beta \in S_n$ with $n > 1$. Prove that if $\alpha \in A_n$, then $\beta\alpha\beta^{-1}$ is in A_n.
4. Write the permutation, in disjoint cycle form, corresponding to (a) R_{90} and (b) V in the left-regular representation of D_4.
5. Let G be a group and H a subgroup. Prove that the relation $g_1 \sim g_2 \iff g_1^{-1}g_2 \in H$ is an equivalence relation on G.

Fun Fact!

In terms of cosets, Number 5 proves that the set of cosets $\{gH \mid g \in G\}$ of H in G partition G! This is because of the practice problem that says $g_1H = g_2H \iff g_1^{-1}g_2 \in H$.