MTH 3110: Abstract Algebra I
Fall Semester 2018
Practice Problem List 7

Prof: Mandi Schaeffer Fry

Just Practice Problems

This homework won't be "due", but I encourage you to work the problems.

1 Before Class On...

- Mon, 11/26: Read Chapter 7
- Mon, 12/3: Instead of class 11/28, you should watch the video I emailed about an introduction to rings. (It is only 25 minutes.) There will be a flash quiz Monday Dec 3 about the video.

2 Notation and Definitions to Know

An $*$ denotes Flashquiz - eligible items for a Flashquiz Monday, 12/3/18.

- *ring (with/without identity - see remark)
- *zero divisor

3 Notes about the video

- The video is from a variation of the abstract algebra course that I taught online last year for in-service teachers, so ignore the reference to "previous videos", etc.
- The video was at an earlier point in the semester, so I apologize for spending so much time on modular arithmetic examples.
- I use matrices as an example and go through how to multiply them. Don't completely ignore this part, but if you haven't seen matrices maybe take this minute or so of the video with a grain of salt.
- IMPORTANT: In the video, I define rings to have a multiplicative identity. In chapter 12 of the book, this would be called a "ring with unity" or "ring with identity", and more generally a ring wouldn't necessarily have a multiplicative identity. This varies book-to-book and mathematician-to-mathematician. Sometimes we joke that a ring without identity should be called "Rng". (Hah!)
- FUN FACT: A ring in which every non-zero element has an inverse is called a *field*.
- FUN FACT: Elements of a ring that have multiplicative inverses are called *units*. This is why we use $U(n)$ to denote set of elements of $\mathbb{Z}_n$ that have multiplicative inverses! It is the "unit group" of $\mathbb{Z}_n$!

4 For Practice...

1. Verify that examples 1-3, 5-7 in Chapter 12 of Gallian (6th ed) are rings as stated.
2. Prove that if a ring has an identity, then it is unique. Prove that if an element of a ring has a multiplicative inverse, then it is unique.
3. For the rings $\mathbb{Z}_6$, $\mathbb{Z}_8$, $\mathbb{Z}_5$, $\mathbb{Z}_7$, under addition and multiplication modulo the appropriate integer, determine (a) all elements with multiplicative inverses and (b) all zero divisors.
4. How did your answers in (3) for $\mathbb{Z}_6, \mathbb{Z}_8$ differ from those for $\mathbb{Z}_5, \mathbb{Z}_7$? Make a conjecture about inverses and zero divisors in $\mathbb{Z}_n$.