

MTH 4110: Abstract Algebra 2 - Spring Semester 2019

Problem List 3

Prof: Mandi Schaeffer Fry

Due 2/15/19

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted without the use of a "full redo". Solutions should be submitted by email to Dr. Mandi (aschae6@msudenver.edu) as a **single PDF file** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!) Recall that if you use LaTeX (including Overleaf) to typeset your homework, you'll get an extra "full redo".

1 Before Class On...

- Mon, 2/11: Read Chapter 14

2 Notation, Definitions, Theorems to Know

An * denotes Flashquiz - eligible items.

- *ideal
- *prime ideal
- *maximal ideal
- *ideal test
- factor ring

3 For Practice...

- Ch. 14, 6th Edition: Problems 7-9, 14, 16, 20, 25, 28, 41

OR

- Ch. 13, 8th Edition: Problems 7-9, 14, 16, 22, 27, 30, 45

The problems to turn in are on the other side

4 To Turn In On 2/15/19

Recall: the term “ideal” means a *two-sided* ideal.

1. Let R be a ring and let I_1 and I_2 be two ideals of R .
 - (a) Prove that $I_1 \cap I_2$ is also an ideal of R .
 - (b) Prove that $I_1 + I_2 := \{x_1 + x_2 \mid x_1 \in I_1, x_2 \in I_2\}$ is also an ideal of R .
2. Let R be a ring with unity and let I be an ideal of R . Prove that if $1 \in I$, then $I = R$.
3. Prove that every ideal in \mathbb{Z} is of the form $\langle a \rangle$ for some $a \in \mathbb{Z}$. (Hint: the division algorithm Thm. 0.1 could be useful.)