MTH 4110: Abstract Algebra 2 - Spring Semester 2019
Problem List 4

Prof: Mandi Schaeffer Fry

Due WEDNESDAY 3/6/19

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted without the use of a "full redo". Solutions should be submitted by email to Dr. Mandi (aschaef6@msudenver.edu) as a **single PDF file** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!) Recall that if you use LaTeX (including Overleaf) to typeset your homework, you'll get an extra "full redo".

1 Before Class On...

- Mon, 2/25: Read Chapter 14
- Mon, 3/4: Read Chapter 15

2 Notation, Definitions, Theorems to Know

An $*$ denotes Flashquiz - eligible items.

- *prime ideal
- *maximal ideal
- factor ring
- *Thm 14.3: relationship between integral domain factor ring and ideals
- *Thm 14.4: relationship between field factor ring and ideals
- *homomorphism of rings
- *kernel
- *first isomorphism theorem
- know properties of homomorphisms (e.g. Thm 15.1, but also Thms 10.1, 10.2)

3 For Practice...

- Ch. 14, 6th Edition: Problems 9, 20, 28, 23, 24, 26, 30, 39, 49, 55, 59;
- Ch. 15, 6th Edition: Problems 1-7, 12-14, 26, 29-34

OR

- Ch. 14, 8th Edition: Problems 9, 22, 30, 25, 26, 28, 32, 41, 53, 59, 63;
- Ch. 15, 8th Edition: Problems 1-6, 10, 14-16, 28, 31-36

The problems to turn in are on the other side

4 To Turn In On 3/6/19

Recall: the term "ideal" means a *two-sided* ideal.

1. Show that $\mathbb{R}[x]/\langle x^2+1\rangle$ is isomorphic to $\mathbb{C}$.
2. Prove that $\langle 2,x,y\rangle$ is maximal in $\mathbb{Z}[x,y]$. Do this by constructing an appropriate homomorphism.
3. Similarly, prove that $\langle x,y\rangle$ is prime but not maximal in $\mathbb{Z}[x,y]$, by constructing an appropriate homomorphism.
4. Let n be an integer with decimal representation $a_k a_{k-1}\cdots a_1 a_0$. (That is, a_i is in the 10^i-th place.) Prove that n is divisible by 3 if and only if $a_k + a_{k-1} + ... + a_1 + a_0$ is divisible by 3. (Follow Example 8, but fill in the missing details!)