MTH 4110: Abstract Algebra 2 - Spring Semester 2019
Problem List 7

Prof: Mandi Schaeffer Fry

Due Friday 4/19/19

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted without the use of a "full redo". Solutions should be submitted by email to Dr. Mandi (aschaef6@msudenver.edu) as a **single PDF file** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!) Recall that if you use LaTeX (including Overleaf) to typeset your homework, you'll get an extra "full redo".

1 Before Class On...

- Wed, 4/10: Finish Reading Chapter 18
- Mon, 4/15: Read Chapter 19

2 Notation, Definitions, Theorems to Know

An $*$ denotes Flashquiz - eligible items.

- *Unique Factorization Domain (UFD)
- *PID $\Rightarrow$ UFD
- *Euclidian Domain (ED)
- *UFD $\Rightarrow$ ED
- vector space
- *linearly independent
- *subspace
- *basis
- *dimension

3 For Practice...

- Ch. 18, 6th Edition: Problems 13, 23, 29, 30, 32, 33, 38
  OR 8th Edition: Problems 13, 23, 33, 34, 36, 37, 42
- Ch. 19, 6th or 8th Edition: Problems 1-10, 19

The problems to turn in are on the other side

4 To Turn In On 4/19/19

1. Show that $\mathbb{Z}$ does not satisfy the "descending chain condition". (That is, show that there exist infinite chains $I_1 \supset I_2 \supset I_3 \supset ...$ of ideals in $\mathbb{Z}$, where the containments are proper.)

2. Show that if D is an integral domain satisfying the descending chain condition (every chain of ideals $I_1 \supset I_2 \supset I_3 \supset ...$ with proper containments is finite), then D is a field.

   (These first two problems correct a hasty non-truth that Dr. Mandi said aloud in class. Minus 5 points from Gryffindor.)

3. Prove or disprove (by providing a counterexample):

   Let $v_1, ..., v_k \in V$ for some vector space V and some $k \geq 2$. If $\{v_1, v_2, ...v_{k-1}\}$ is a linearly dependent set, then $\{v_1, .., v_k\}$ is also linearly dependent.

4. Prove or disprove (by providing a counterexample):

   Let V be a vector space of dimension 3 and let $v_1, v_2, v_3 \in V$. If v_3 is not a linear combination of v_1 and v_2, then $\{v_1, v_2, v_3\}$ is a linearly independent set.