MTH 4110: Abstract Algebra 2 - Spring Semester 2019
Problem List 8

Prof: Mandi Schaeffer Fry

Due Friday 4/26/19

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted without the use of a "full redo". Solutions should be submitted by email to Dr. Mandi (aschaef6@msudenver.edu) as a **single PDF file** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!) Recall that if you use LaTeX (including Overleaf) to typeset your homework, you'll get an extra "full redo".

1 Before Class On...

- Mon, 4/22: Read Chapter 20
- Mon, 4/29: Read Chapter 21
- Wed, 5/1: Read Chapter 22
- Mon, 5/6: Read Chapter 32 (BOOM! WE MADE IT!)

2 Notation, Definitions, Theorems to Know

An $*$ denotes Flashquiz - eligible items.

- *extension field
- *index of an extension
- *$F(a_1, ..., a_n)$
- *splitting field
- *separable polynomial
- Statement of the theorem saying $F(a) \cong F[x]/\langle p(x) \rangle$
- Statement that splitting fields are unique

3 For Practice...

- Ch. 20, 6th or 8th Edition: Problems 1-5, 13, 18-21, 28

The problems to turn in are on the other side

4 To Turn In On 4/26/19

1. Let $a, b \in \mathbb{Q}$. Prove that $\mathbb{Q}(\sqrt{a}, \sqrt{b}) = \mathbb{Q}(\sqrt{a} + \sqrt{b})$. (Hint: Consider $\dfrac{1}{\sqrt{a} + \sqrt{b}}$.)

2. Let $a \in \mathbb{Q}$. Determine all subfields of $\mathbb{Q}(\sqrt{a})$. Prove your answer. (Note: you might have cases!)

3. Let $F = \mathbb{Z}_2$ and let $f(x) = x^3 + x + 1 \in F[x]$.

   (a) Show that $f(x)$ is irreducible over F.

   (b) Let a be a root of $f(x)$ in some extension of F. Write each element of $F(a)$ in terms of a, and write out the complete multiplication table for $F(a)$.

   (c) Show that a^2 and $a^2 + a$ are zeros of $x^3 + x + 1$ in $F(a)$.