MTH 4110: Abstract Algebra 2 - Spring Semester 2019
Problem List 9

Prof: Mandi Schaeffer Fry

Due Monday 5/06/19

You are encouraged to ask questions during office hours. You are also encouraged to work through problems together and bounce ideas off of one another; however, the actual write up should be done on your own. This means your homework should not be identical to another person's.

NOTE: Late homework will NOT be accepted without the use of a "full redo". Solutions should be submitted by email to Dr. Mandi (aschaef6@msudenver.edu) as a **single PDF file** in clear writing, written neatly, using complete sentences. (This may require re-writing your final draft to turn in!) Recall that if you use LaTeX (including Overleaf) to typeset your homework, you'll get an extra "full redo".

1 Before Class On...

- Mon, 4/29: Read Chapter 21
- Wed, 5/1: Read Chapter 22
- Mon, 5/6: Read Chapter 32 (BOOM! WE MADE IT!)

2 Notation, Definitions, Theorems to Know

An $*$ denotes Flashquiz - eligible items.

- *extension field
- *degree of an extension
- *$F(a_1, ..., a_n)$
- *splitting field
- *separable polynomial
- Statement of the theorem saying $F(a) \cong F[x]/\langle p(x)\rangle$
- Statement that splitting fields are unique
- *algebraic elements, extensions
- *transcendental elements, extensions
- statement that $[K : F] = [K : E][E : F]$
- statement that finite implies algebraic, and its proof
- statement about the structure of finite fields
- statement that for each power of a prime, there is a unique field of that size

3 For Practice...

- Ch. 20, 6th or 8th Edition: Problems 1-5, 13, 18-21, 28
- Ch. 21, 6th or 8th Edition: Problems 1, 3, 5, 8, 9, 14, 15, 17
- Ch. 22, 6th Edition: Problems 1-7, 21, 27, 31, 33 OR 8th Ed: Problems 1,2,7,8,9, 10, 11, 27, 33, 37, 39

The problems to turn in are on the other side

4 To Turn In On 5/06/19

1. In each of the following, find a polynomial $p(x)$ such that $\mathbb{Q}(a) \cong \mathbb{Q}[x]/\langle p(x)\rangle$. Prove your answers.
   (a) $a = \sqrt{5}$
   (b) $a = \sqrt{3+\sqrt{5}}$

2. (a) Show that $\mathbb{Q}(\sqrt{5}) \subseteq \mathbb{Q}(\sqrt{3+\sqrt{5}})$.
   (b) Find a polynomial $p(x)$ such that $\mathbb{Q}(\sqrt{3+\sqrt{5}}) \cong \mathbb{Q}(\sqrt{5})[x]/\langle p(x)\rangle$. Prove your answer.
   (c) Determine each of the following (and, you guessed it, prove your answer):
      i. $[\mathbb{Q}(\sqrt{5}) : \mathbb{Q}]$
      ii. $[\mathbb{Q}(\sqrt{3+\sqrt{5}}) : \mathbb{Q}]$
      iii. $[\mathbb{Q}(\sqrt{3+\sqrt{5}}) : \mathbb{Q}(\sqrt{5})]$

3. (a) Show that $\mathbb{Q}(\sqrt{3-\sqrt{5}}) = \mathbb{Q}(\sqrt{3+\sqrt{5}})$. Why does this imply that $\mathbb{Q}(\sqrt{3+\sqrt{5}})$ is a splitting field for the polynomial you found for 1(b)?
   (b) However, show that $\mathbb{Q}(\sqrt{2-\sqrt{5}}) \neq \mathbb{Q}(\sqrt{2+\sqrt{5}})$.

4. Show that there are exactly two automorphisms of the field $\mathbb{Q}(\sqrt{5})$. (Hint: You may use the fact that you proved...well, maybe...on the exam, which states that any automorphism of a field containing $\mathbb{Q}$ must act as the identity map on $\mathbb{Q}$.)