Probability and Statistics

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Notes

Topics

1 Cumulative Distribution Functions (Cont'd)

Notes

Objectives

Objectives:

- Obtain percentiles from the cumulative distribution function of a continuous random variable.

Notes

Cumulative Distribution Functions (Cont'd) (4.2)

Properties of Cumulative Distribution Functions:

1. $F(x)$ is non-decreasing.
2. $\lim_{x\to\infty} F(x) = 1$.
3. $\lim_{x\to-\infty} F(x) = 0$.
4. If X is *continuous* with pdf $f(x)$, then $F(x)$ is also continuous.
5. If X is *discrete* with pmf $p(x)$, then $F(x)$ is a (right-continuous) step function, with steps of size $p(x)$ at each of the possible values x of X.

Notes

Percentiles of Continuous Distributions (4.2)

- For $0 < p < 1$, the $\mathbf{100}\boldsymbol{p}$***th percentile*** of the distribution of a **continuous** random variable X is the value η defined by

$$P(X \le \eta) \;=\; p.$$

Examples:
- The **90th percentile** of scores on the verbal SAT is $\eta = 600$, and is the score below which **90%** of all scores lie.
- The **99th percentile** of U.S. incomes is $\eta = \$400,000$ (according to a 2012 CNN news report), and is the income below which **99%** of all incomes lie.

Notes

- Because η has to satisfy

$$P(X \le \eta) \;=\; p\,,$$

we have a convenient way to find the value of η if we know the **cdf** of X.

Finding Percentiles: The $\mathbf{100}\boldsymbol{p}$**th percentile** of the distribution of a continuous random variable X whose **cdf** is $F(x)$ is obtained by solving

$$F(\eta) \;=\; p$$

for η.

Notes

Example

Let

X = A company's profit (in millions of dollars) in the coming year

Suppose the **pdf** of X is

$$f(x) = \begin{cases} \frac{3}{x^4} & \text{for } x \ge 1 \\ 0 & \text{for } x < 1 \end{cases}$$

so the **cdf** of X is

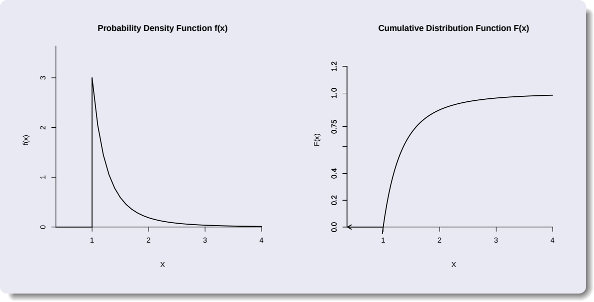
$$F(x) = \begin{cases} 1 - \frac{1}{x^3} & \text{for } x \ge 1 \\ 0 & \text{for } x < 1 \end{cases}$$

Notes

The **75th percentile** of the distribution of X is the value marked η in the graphs on the next slide.

η is the profit level that will be exceeded with probability 0.25.

Notes



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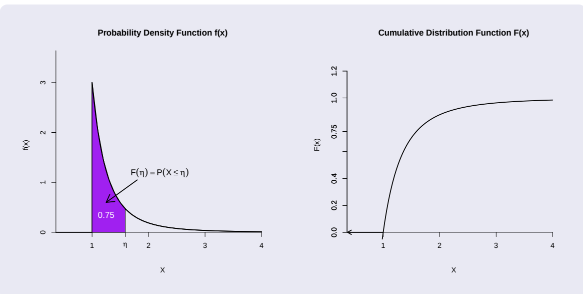

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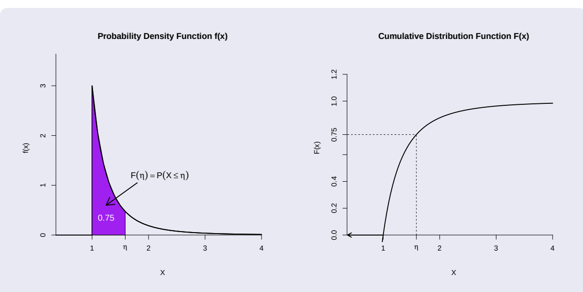

Notes

To solve for $\boldsymbol{\eta}$ (the **75th percentile**), set

$$F(\eta) \;=\; 0.75$$

and solve for η.

This gives

$$\begin{aligned} 1 - \frac{1}{\eta^3} &= 0.75 \\ \Rightarrow \qquad \eta^3 &= \frac{1}{0.25} \\ \Rightarrow \qquad \eta &= \left(\frac{1}{0.25}\right)^{1/3} \;=\; \mathbf{1.59}. \end{aligned}$$

Notes