Probability and Statistics

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Notes

Topics

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Objectives

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- Produce and interpret a scatterplot of bivariate numerical data.
- Compute and interpret the sample correlation between two numerical variables.

Notes

Scatterplots (12.1)

- ***Bivariate data*** consist of observations of **two variables** on each individual.
- Bivariate data can be used to investigate the **relationship** between the variables.
- The two variables usually play different roles:

  One of them, the ***explanatory*** (or ***independent***) variable, ”explains” variation in the other, which is called the ***response*** (or ***dependent***) variable.

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Example

Consider a study of the **relationship** between **time spent studying** for an exam and the **exam score**.

The **explanatory variable** is **time spent studying** and the **response** is the **exam score**.

Notes

Example

Here are **lengths** (cm) and **weights** (g) of $n = 9$ female snakes.

Snake	Length	Weight
1	60	136
2	69	198
3	66	194
4	64	140
5	54	93
6	67	172
7	59	116
8	65	174
9	63	145

Notes

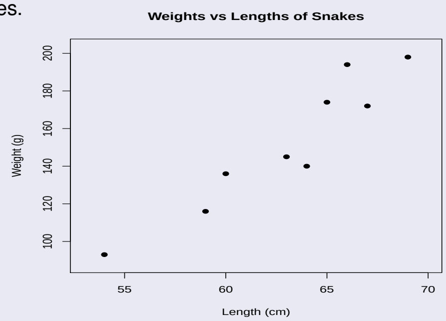
The **explanatory variable** is **length** and the **response** is **weight**.

Notes

- ***Scatterplot of Bivariate Numerical Data***: Shows the **relationship** between the two variables.
  - Plot each **bivariate observation** as a point, with the explanatory variable as the x-coordinate and the response as the y-coordinate.

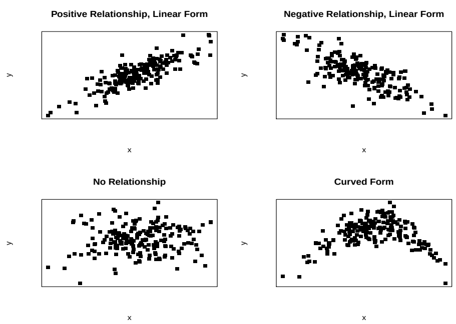
Example (Cont'd)

Here's the **scatterplot** of the **lengths** and **weights** of the snakes.



Notes

The figures below illustrate some common **scatterplot patterns**.



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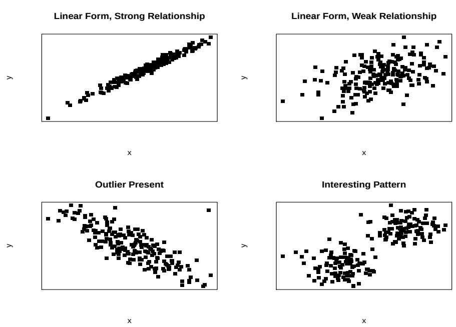

Notes

- **Terminology** used to describe **scatterplot patterns**:
  1. ***Form*** of the pattern (i.e. is it ***linear***, ***curved***, etc.?).
  2. The ***direction*** of the relationship between the two variables:
     - ***Positive***: Y tends to be *large* when X is *large* and *small* when X is *small* (the points in the plot slope upward to the right).
     - ***Negative***: Y tends to be *small* when X is *large* and *large* when X is *small* (the points in the plot slope downward to the right).
  3. The ***strength*** of the relationship (i.e. how distinct is the pattern?)
  4. ***Outliers*** or other ***interesting features***.

Notes

Correlation (12.5)

- **Notation** for a data set of n bivariate observations:

Observation	Explanatory Variable	Response Variable
1	x_1	y_1
2	x_2	y_2
3	x_3	y_3
$\vdots$	$\vdots$	$\vdots$
n	x_n	y_n

Notes

- When two variables exhibit (approximately) a **linear relationship**, we summarize that relationship by the ***sample correlation***, denoted r, defined as follows.

Correlation: The correlation between two variables

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{s_x}\right)\left(\frac{y_i-\bar{y}}{s_y}\right) = \frac{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{s_x s_y}$$

where $\bar{x}$ and $\bar{y}$ are the sample means of the x_i's and y_i's, respectively, and s_x and s_y are their sample standard deviations.

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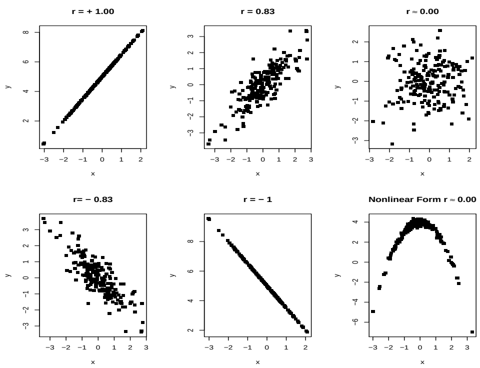

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- The following **properties of the correlation** r help us interpret its value:
  1. The value of r will always lie **between -1.0 and 1.0**.
  2. The **sign** of r tells us the **direction** of the relationship between X and Y:
     - Positive r values indicate a **positive** relationship.
     - Negative r values indicate a **negative** relationship.

Notes

3. The value of r also tells us how **strong** the relationship between X and Y is:
   - r values **near zero** imply a very **weak** relationship or none at all.
   - r values **close to -1.0** or **1.0** imply a very **strong** linear relationship.
   - The extreme values $r = -1.0$ and $r = 1.0$ occur only when there's a **perfect linear** relationship.
4. The value of r doesn't depend on which variable is labeled X and which is labeled Y.

Notes

5. r has no units of measure (e.g. it's not measured in inches or pounds or dollars, even if the data are measured such units).
6. The value of the r is **unaffected** by a (linear) **change of measurement scale** of either X or Y (e.g. converting from Celsius to Fahrenheit).
7. r only measures the strength of the **linear relationship** between X and Y. In particular, curved relationships often have r near zero.
8. r is **not resistant** to outliers.

Notes

Example (Cont'd)

Here are the summary statistics for the **lengths** and **weights** of the $n = 9$ snakes:

	Lengths	Weights
Mean	$\bar{x} = 63.00$	$\bar{y} = 152.00$
Standard Deviation	$s_x = 4.64$	$s_y = 35.34$

Compute the correlation between length and weight and interpret the result.

Notes

The **correlation** between **length** and **weight** is

$$
\begin{aligned}
r &= \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{s_x}\right)\left(\frac{y_i-\bar{y}}{s_y}\right) \\
&= \frac{1}{9-1}\left[\left(\frac{60-63}{4.64}\right)\left(\frac{136-152}{35.34}\right)+\left(\frac{69-63}{4.64}\right)\left(\frac{198-152}{35.34}\right)\right. \\
&\quad \left. + \cdots + \left(\frac{63-63}{4.64}\right)\left(\frac{145-152}{35.34}\right)\right] \\
&= \mathbf{0.944},
\end{aligned}
$$

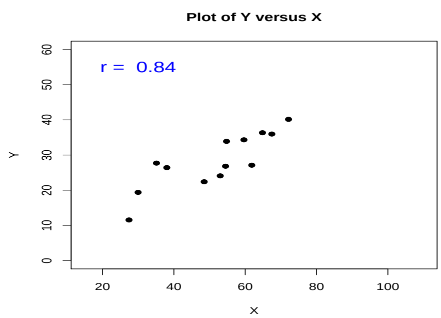
which is consistent with the **strong**, **positive** linear relationship seen in the scatterplot.

Notes

- The next plots show that the **correlation** r is **not resistant** to outliers.

  The location of the outlier in the scatterplot, relative to the rest of the data, determines the affect that the outlier has on the correlation.

Notes



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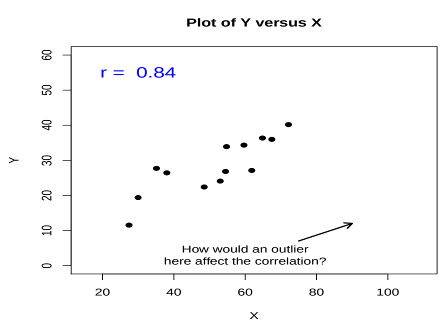

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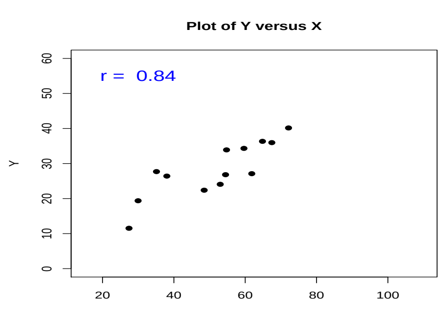

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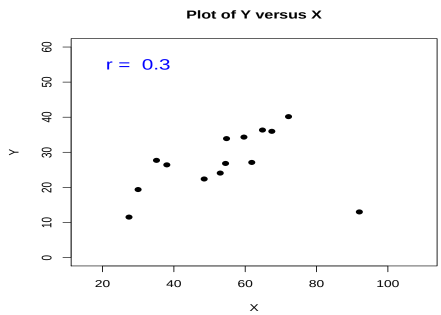

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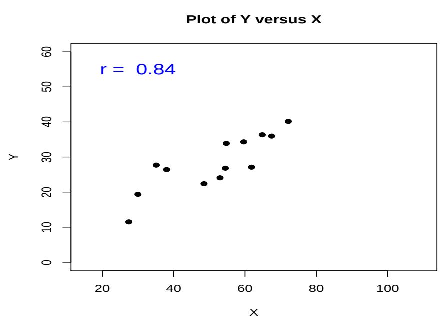

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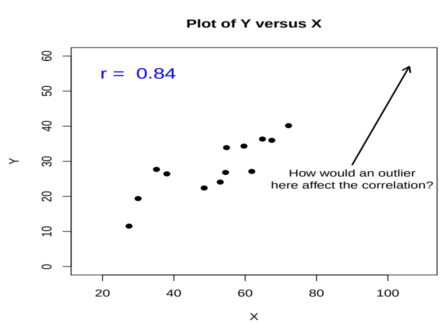

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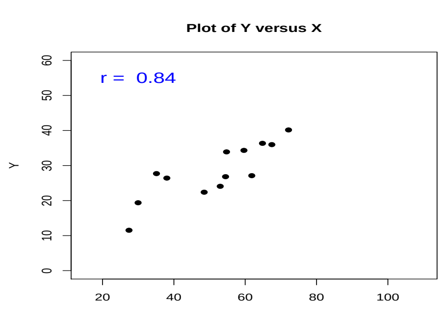

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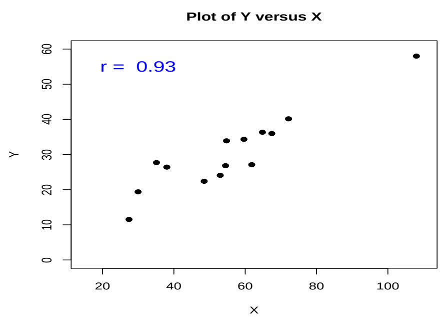

Notes

- A **correlation** between two variables, even if it's very strong, **doesn't imply a cause-and-effect relationship**.

  The relationship might instead be the result of one or more **confounding variables** "lurking" in the background (i.e. not measured).

  A ***confounding variable*** is a variable that's related to both X and Y. As the confounding variable changes, X and Y simultaneously change.

- The next two examples illustrate the notion of **confounding variables**.

Notes

Example

Data on the **number of TV sets per capita** and the **average life expectancy** for each of the world's nations shows a **strong positive correlation** between these two variables – nations with more TV sets have longer life expectancies.

Can we conclude that owning more TVs **causes** people to live longer? If not, what's the main **confounding variable**?

Here **wealth** is a **confounding variable** – nations with more TVs are wealthier, and wealth influences life expectancies (via more nutritious diets, better hospitals, etc.).

Notes

Example

A 1998 NIH study found that people aged 65 or older who **attend church** more often have lower incidences of high **blood pressure** than those who attend church less often.

An article about the study in *USA Today* (Aug. 11, 1998) stated:

> *"Attending religious services lowers blood pressure."*

This implies a **cause-and-effect** relationship between **church attendance** and **blood pressure**.

Is it reasonable to draw such a conclusion from the study?

Notes

It's known that **smoking** cigarettes and **drinking** alcohol can increase **blood pressure**, and people who **attend church** regularly may be **less likely** than others to **smoke** or **drink**.

Therefore **smoking** and **drinking** are possible **confounding variables** that may explain the observed relationship between **church attendance** and **blood pressure**.

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