Solutions to 2026 AP Calculus AB Free Response Questions

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1. Unless otherwise indicated, values of $M(t)$ used in the following are obtained from the table given in the problem.

   A. The average rate of change of $M(t)$ over the interval $5 \le t \le 10$ is, in birds per day per day,

   $$\frac{M(10) - M(5)}{10 - 5} = \frac{16 - 7}{5} = \frac{9}{5}. \tag{1}$$

   B. i. The midpoint sum for $\int_0^{30} M(t)\,dt$ with the three subintervals $[0, 10]$, $[10, 20]$, and $[20, 30]$ is

   $$M(5)(10 - 0) + M(15)(20 - 10) + M(25)(30 - 20) \tag{2}$$

   $$= 7 \cdot 10 + 6 \cdot 10 + 2 \cdot 10 \tag{3}$$
   $$= 70 + 60 + 20 \tag{4}$$
   $$= 150. \tag{5}$$

   ii. $M(t_0)$ denotes the rate at which male birds arrive at time t_0, so $\int_0^{30} M(t)\,dt$ gives the total number of birds that arrive during the thirty-day period $0 \le t \le 30$.

   C. We are given

   $$F(t) = \begin{cases} 0 & \text{for } 0 \le t < 15 \\ 18 + 16 \sin\left(\frac{\pi}{20}(t + 15)\right) & \text{for } 15 \le t \le 45 \end{cases} \tag{6}$$

as the rate at which female birds arrive during the period $15 \leq t \leq 45$. Thus, during the period $15 \leq t \leq 45$, the total number of female birds that arrive is

$$\int_{15}^{45} F(t)\, dt = \int_{15}^{45} \left[18 + 16 \sin\left(\frac{\pi}{20}(t + 15)\right)\right]\, dt \tag{7}$$

(8)

Numeric integration gives

$$\int_{15}^{45} F(t)\, dt \sim 641.859, \tag{9}$$

or, to the nearest integer, 642.

D. We easily calculate that $D(15) = 4$, while $D(20) \sim -1.686$. The function M is given differentiable, and the function F is surely differentiable. Both are therefore continuous, and $D(t) = M(t) - F(t)$ is therefore continuous. The Intermediate Value Theorem for Continuous Functions therefore guarantees that there is a value ξ in the interval $15 < t < 20$ where $D(\xi) = 0$.

2. A. The area of the region R is

$$\int_0^1 g(x)\,dx = \int_0^1 \frac{14x+12}{x+12}\,dx \tag{10}$$

$$\sim 1.513338 \sim 1.513, \tag{11}$$

where we have integrated numerically.

Note The integral can be done symbolically without much difficulty:

$$\int_0^1 \frac{14x+12}{x+12}\,dx = \int_0^1 \left[14 - \frac{156}{x+12}\right]\,dx \tag{12}$$

$$= \left[14x - 156\ln(x+12)\right]\Big|_0^1 \tag{13}$$

$$= 14 - 156\ln\frac{13}{12}. \tag{14}$$

B. The required integral is

$$\frac{1}{3}\int_0^1 [g(x)]^2\,dx = \frac{1}{3}\int_0^1 \left[\frac{14x+12}{x+12}\right]^2\,dx. \tag{15}$$

C. Solving the equation $f(a) = g(a)$ numerically for $a \neq 1$, we find that $a \sim 3.2558165$. Thus, we seek

$$A = \int_0^1 \left[f(x) - g(x)\right]\,dx + \int_1^a \left[g(x) - f(x)\right]\,dx \tag{16}$$

$$\tag{17}$$

Now, numerical integrations give

$$\int_0^1 \left[f(x) - g(x)\right]\,dx = \int_0^1 \left[(1.43^x + 0.57) - \frac{14x+12}{x+12}\right]\,dx \tag{18}$$

$$\sim 0.37291 \tag{19}$$

and

$$\int_1^a \left[g(x) - f(x)\right]\,dx = \int_1^a \left[\frac{14x+12}{x+12} - (1.43^x + 0.57)\right]\,dx \tag{20}$$

$$\sim 0.25887. \tag{21}$$

Thus, to three decimal places,

$$A = \sim 0.632. \tag{22}$$

D. The required integral is

$$\pi\int_1^{7/2} [h(y)]^2\,dy = \pi\int_1^{7/2} \left[\frac{12y-12}{14-y}\right]^2\,dy. \tag{23}$$

3. A. The slope field shown can't be the slope field for the differential equation

$$\frac{dH}{dt} = -\frac{1}{15}(H - 20) \tag{24}$$

because when $H > 20$ the equation (24) tells us that $\frac{dH}{dt}$ is negative, while all the slopes that appear in the given slope field are positive—even though $H > 20$ throughout the region that appears.

B. We are given $H(0) = 75$. Thus, when $t = 0$ we have

$$H'(0) = -\frac{1}{15}[H(0) - 20] \tag{25}$$

$$= -\frac{1}{15}(75 - 20) \tag{26}$$

$$= -\frac{55}{20} = -\frac{11}{4}. \tag{27}$$

This is the slope of the line tangent to the graph of H at time $t = 0$.

C. If (24) holds, if $H'' = \frac{1}{225}(H-20)$, and if the linearization of H at $t = 0$ is used to approximate $H(5)$, we expect that the approximation will underestimate $H(5)$. This is because we know that $H(0) = 75$, so that $H''(0) = \frac{1}{225}[H(0) - 20] = \frac{11}{45} > 0$, making the curve concave upward near $t = 0$. Thus, the tangent line through $\big(0, H(0)\big)$ lies below the $H(t)$ curve in that region.

Note I believe this to be the argument the readers expect to see. It's a shaky one, because we have no information about how rapidly $H''(t)$ changes, and the interval over which we make the linearization approximation is fairly large. From what we know at this point, we should consider the possibility that the second derivative becomes negative, so that the curve turns downward—possibly even crossing the tangent line. The only way I see to do this is to use the solution that we find in the next part of the problem—and we can then calculate directly whether the linearization gives an over estimate or an underestimate. This seems to me to be a poorly posed problem.

D. If $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$ and $H(0) = 75$, then

$$\frac{dH}{H - 20} = -\frac{1}{15}\,dt, \tag{28}$$

so that

$$\int_{75}^{H} \frac{dh}{h-20} = -\frac{1}{15}\int_{0}^{t} d\tau \tag{29}$$

$$\ln(h-20)\bigg|_{75}^{H} = -\frac{1}{15}\tau\bigg|_{0}^{t} \tag{30}$$

$$\frac{H-20}{55} = e^{-t/15} \tag{31}$$

$$H = 20 + 55e^{-t/15}. \tag{32}$$

4. A. If $g(x) = f(x) - \ln x$ for $x > 0$, then

$$g'(x) = f'(x) - \frac{1}{x}, \tag{33}$$

so that

$$g'(2) = f'(2) - \frac{1}{2} = 1.5 - \frac{1}{2} = 1, \tag{34}$$

it being given that $f'(2) = 1.5$.

B. The graph of f has a point of inflection at every point where f' has a local extremum—that is, at points where the monotonicity of f' changes. From the given graph, we find such points at $x = -3$, at $x = 1$, and at $x = 3$.

C. The graph of f is increasing on intervals where $f'(x) > 0$, decreasing on intervals where $f'(x) < 0$. The graph of f is concave upward on intervals where $f'(x)$ is increasing, concave downward on intervals where $f'(x)$ is decreasing. From the graph of f', we see that f decreasing and concave downward on $(-4, -3)$, decreasing and concave upward on $-3, -2)$, increasing and concave upward on $(-2, 1)$ and on $(3, 4)$, and *increasing but concave downward on* $(1, 3)$.

D. The function f has a critical point at $x = -2$ where $f'(x)$ is negative to the left but positive to the right—making this point a relative minimum. It has another critical point at $x = 3$, where $f'(x)$ is positive to both sides—making this point a saddle point, which can be neither a minimum nor a maximum. There can be no minimum at the left endpoint, $x = -4$, because $f'(x) < 0$ for values of x immediately to the right of of $x = -4$—meaning that f decreases just to the right of $x = -4$. Similarly, there can be no minimum at the right endpoint $x = 4$, because $f'(x) > 0$ for values of x immediately to the left $x = 4$. The absolute minimum guaranteed by the Extreme Value Theorem (which applies because f is differentiable and therefore continuous) must occur at either a critical point or an end point, so the absolute minimum value of $f(x)$ on $[-4, 4]$ occurs at $x = -2$.

The absolute maximum guaranteed by the Extreme Value Function must occur at one of the endpoints, $x = -4$ or $x = 4$, because all of the critical points are accounted for.

We have, by the Fundamental Theorem of Calculus,

$$f(4) = f(-4) + \int_{-4}^{4} f'(x)\, dx. \tag{35}$$

Let g be the function defined on $[-4, 4]$ as the function whose graph is the union of the line segment connecting the point $(-4, -1)$ and the point $(-2, -1)$, the line segment connecting the point $(-2, 0)$ and the point $(1, 3)$, the open line segment connecting the point $(1, 3)$ and the point $(2, 0)$, and, finally, the open-closed line segment connecting the point $(2, 0)$ with the point $(4, 0)$. More formally,

$$g(x) = \begin{cases} -1 & -4 \leq x < -2 \\ 2 + x & -2 \leq x \leq 1 \\ 6 - 3x & 1 < x < 2 \\ 0 & 2 \leq x \leq 4 \end{cases} . \tag{36}$$

It is clear from the graph that $f'(x) \geq g(x)$ for all $x \in [-4, 4]$. Consequently,

$$\begin{aligned} f(4) &= f(-4) + \int_{-4}^{4} f'(x)\,dx && (37) \\ &\geq f(-4) + \int_{-4}^{4} g(x)\,dx && (38) \\ &\geq f(-4) + \int_{-4}^{-2} (-1)\,dx + \int_{-2}^{1} (2 + x)\,dx + \int_{1}^{2} (6 - 3x)\,dx + \int_{2}^{4} 0\,dx && (39) \\ &\geq f(-4) + (-2) + \frac{1}{2} \cdot 3 \cdot 3 + \frac{1}{2} \cdot 1 \cdot 3 + 0 && (40) \\ &\geq f(-4) + 4. && (41) \end{aligned}$$

Therefore the absolute maximum for f must be at $x = 4$, because we have shown that $f(4) > f(-4)$.

5. We are given

$$v(t) = \begin{cases} t^4 - 8t^3 + 16t^2 & \text{for } 0 \le t \le 64 \\ 0 & \text{for } 4 < t < 6 \\ 10\cos\left(\frac{\pi}{3}t\right) - 10 & \text{for } 6 \le t \le 12 \end{cases}. \tag{42}$$

A. Acceleration is the time derivative of velocity. When $0 \le t \le 4$,

$$v'(t) = \frac{d}{dt}(t^4 - 8t^3 + 16t^2) \tag{43}$$

$$= 4t^3 - 24t^2 + 32t, \tag{44}$$

so that

$$v'(1) = 4 \cdot 1^3 - 24 \cdot 1^2 + 32 \cdot 1 = 12. \tag{45}$$

B. We see from the graph that velocity is both positive and increasing on the interval $0 < t < 2$, so the car is speeding up when $t = 1$. More formally, writing $s(t)$ for speed at time t, we have $s(t) = |v(t)|$, whence $s(t) \ge 0$. Moreover

$$\big(s(t)\big)^2 = \big(v(t)\big)^2 \tag{46}$$

so that

$$2s(t)s'(t) = 2v(t)v'(t). \tag{47}$$

Because $s(t) \ge 0$ by definition, we infer that $s'(t) > 0$ when, and only when, $v(t)$ and $v'(t)$ both have the same sign. We have seen that $v'(1) = 12 > 0$; we compute that

$$v(1) = 1^4 - 8 \cdot 1^3 + 16 \cdot 1^2 = 9 > 0, \tag{48}$$

from which we conclude that $s'(t) > 0$—meaning that, $s'(t)$, being a continuous function, $s(t)$ is increasing near $t = 1$. Thus, the car is speeding up when $t = 1$.

C. Distance traveled during a time interval I is the time integral of speed over I. In the interval $0 \le t \le 4$, velocity is a positive real number and is therefore identical to speed. Thus, if D is the distance, in feet, that this car travels during the interval $0 \le t \le 4$, then

$$D = \int_0^4 v(t)\,dt \tag{49}$$

$$= \int_0^4 \big(t^4 - 8t^3 + 16t^2\big)\,dt \tag{50}$$

$$= \left(\frac{t^5}{5} - 2t^4 + \frac{16}{3}t^3\right)\Bigg|_0^4 = \frac{512}{15}. \tag{51}$$

The car travels $\frac{512}{15}$ feet when $0 \leq t \leq 4$.

D. The average velocity of the car over the interval $6 \leq t \leq 12$, where

$$v(t) = 10\cos\left(\frac{\pi}{3}t\right) - 10 \tag{52}$$

is

$$\frac{1}{12-6}\int_6^{12} v(t)\,dt = \frac{1}{6}\int_6^{12}\left[10\cos\left(\frac{\pi}{3}t\right) - 10\right]\,dt \tag{53}$$

$$= \frac{10}{3}\left(\frac{3}{2\pi}\sin\frac{\pi t}{3} - \frac{t}{2}\right)\Big|_6^{12} \tag{54}$$

$$= -20 - (-10) = -10. \tag{55}$$

Average velocity of the car over the time interval $6 \leq t \leq 12$ is -10 feet per second.

6. A. The function f is given (twice) differentiable, and therefore continuous,. Consequently,

$$\lim_{x \to 2} f(x) = f(2) = 3, \tag{56}$$

and it follows (because the limit in the denominator is non-zero) that

$$\lim_{x \to 2} \frac{f(x)}{x} = \frac{\lim_{x \to 2} f(x)}{\lim_{x \to 2} x} \tag{57}$$

$$= \frac{3}{2}. \tag{58}$$

(a) B. If $g(x) = f[f(x)]$, then the Chain Rule gives

$$g'(x) = f'[f(x)] \cdot f'(x). \tag{59}$$

Thus

$$g'(2) = f'[f(2)] \cdot f'(2) \tag{60}$$

$$= f'[3] \cdot 4 = 9 \cdot 4 = 36. \tag{61}$$

C. By the Fundamental Theorem of Calculus,

$$h(x) = h(0) + \int_0^x f'(3t)\, dt. \tag{62}$$

We let $u = 3t$, then $du = 3\, dt$ so that $dt = \frac{1}{3}\, du$. Moreover, $u = 0$ when $t = 0$, and $u = 3x$ when $t = x$. Thus

$$\int_0^x f'(3t)\, dt = \frac{1}{3} \int_0^{3x} f'(u)\, du \tag{63}$$

$$= \frac{1}{3}\left[f(3x) - f(0)\right], \tag{64}$$

so that

$$h(x) = 10 + \frac{1}{3}\left[f(3x) + 1\right]. \tag{65}$$

Thus,

$$h(2) = 10 + \frac{1}{3}\left[f(6) + 1\right] \tag{66}$$

$$= 10 + \frac{1}{3}(5 + 1) = 12. \tag{67}$$

D. If

$$k(x) = \int_0^x t^2 f(t)\,dt, \tag{68}$$

then, by the Fundamental Theorem of Calculus,

$$k'(x) = \frac{d}{dx}\left[\int_0^x t^2 f(t)\,dt\right] \tag{69}$$
$$= x^2 f(x). \tag{70}$$

and

$$k''(x) = \frac{d}{dx}\left[x^2 f(x)\right] \tag{71}$$
$$= 2xf(x) + x^2 f'(x). \tag{72}$$

Thus,

$$k''(3) = 2 \cdot 3 \cdot f(3) + 3^2 \cdot f'(3) \tag{73}$$
$$= 2 \cdot 3 \cdot 8 + 9 \cdot 9 = 129. \tag{74}$$