Physics 2331-002  
Final Exam  
December 10, 2013  

Instructions: This exam is worth a total of 100 points and each problem is worth 10 points. Do at least 10 of the following 12 problems. Show your work to be eligible for partial credit. Remember to be careful about significant figures, units and indicating directions where appropriate. Good luck!

1. What is the magnitude of an electric field in which the electric force on a proton is equal in magnitude to its weight?

\[ E = \frac{mg}{q} \]

\[ E = \frac{1.64 \times 10^{-24} \text{ N}}{1.6 \times 10^{-7} \text{ N/C}} \]

\[ = 1.02 \times 10^{-7} \text{ N/C} \]

2. In one example, we discovered that using Gauss's Law the electric field due to an infinite line of charge perpendicular to the line has a magnitude of

\[ E = \frac{\lambda}{2\pi\varepsilon_0 r} \]

For an imaginary cylinder with radius \( r = 0.250 \text{ m} \) and \( l = 0.400 \text{ m} \) with an infinite line of positive charge running along its axis. The charge per unit length on the line is \( \lambda = 3.00 \ \mu\text{C/m} \).

a. What is the electric flux through the cylinder due to this electric charge?

\[ \Phi_E = E \cdot A \]

\[ \Phi_E = \left( \frac{\lambda}{2\pi\varepsilon_0 r} \right) \left( 2\pi r l \right) \]

\[ = \frac{\lambda l}{\varepsilon_0} \]

\[ = \frac{3.00 \times 10^9 \text{ C/m} \ (0.400 \text{ m})}{8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^2} \]

\[ = 1.36 \times 10^5 \text{ Nm}^2/\text{C} \]

b. If the radius of the imaginary cylinder is reduced to 0.125 m, what is the electric flux passing through it?

The flux is the same as in part a.

In the expression above, the \( r \)-term cancels.

In addition, the \( E \) is constant.

3. An electron is to be accelerated from \( 3.00 \times 10^6 \text{ m/s} \) to \( 8.00 \times 10^6 \text{ m/s} \).

Through what potential difference must the electron pass to accomplish this task?

\[ q \varepsilon_0 V = \frac{m}{2} \left( V_f^2 - V_i^2 \right) \]

\[ q \varepsilon_0 V = \frac{9.11 \times 10^{-31} \text{ kg}}{2 \left( 1.60 \times 10^{-19} \text{ C} \right)} \sqrt{(8.00 \times 10^6 \text{ m/s})^2 - (6.00 \times 10^6 \text{ m/s})^2} \]

\[ = 71.2 \text{ V} \]

\[ \varepsilon_0 V = \frac{m}{2q} \left( V_f^2 - V_i^2 \right) \]

\[ = \frac{157 \text{ Volts}}{} \]
4. Assume the each capacitor in the network shown below has a value of 1.00 \( \mu \text{F}. \) Find the equivalent capacitance between points a and b.

\[
\frac{1}{C_{eq}} = \frac{1}{1.0 \mu \text{F}} + \frac{1}{1.0 \mu \text{F}} + \frac{1}{1.0 \mu \text{F}} = \frac{3}{1.0 \mu \text{F}}
\]

\[C_{eq} = \frac{1}{3} \mu \text{F}\]

\[
C_{total} = 1 \mu \text{F} + \frac{1}{3} \mu \text{F} + \frac{1}{3} \mu \text{F}
\]

\[C_{total} = 1 \frac{2}{3} \mu \text{F} = 1.67 \mu \text{F}\]

5. Assume the each resistor in the network below has a value of 100.0 \( \Omega \). What is the equivalent resistance between points a and b?

\[
\frac{1}{\text{Reg}} = \frac{1}{200 \Omega} + \frac{1}{200 \Omega} + \frac{1}{100 \Omega} = \frac{4}{200 \Omega}
\]

\[\text{Reg} = 50.00 \Omega\]

6. A singly charged ion of \(^7\text{Li}\) (an isotope of lithium) has a mass of \(1.16 \times 10^{-26} \text{ kg}\). It is accelerated through a potential difference of 220 V and then enters a magnetic field with magnitude 0.723 T perpendicular to the path of the ion. What is the ion's path in the magnetic field?

\[
r = \frac{mv}{19B}
\]

\[\omega = \frac{19B}{m}
\]

\[v = \sqrt{\frac{2qV}{m}}
\]

\[r = \frac{mv}{19B} = \frac{(1.16 \times 10^{-26} \text{ kg}) (7.79 \times 10^4 \text{ m/s})}{1.60 \times 10^{-19} \text{ C} (0.723 \text{ T})} = 7.81 \times 10^5 \text{ m}
\]

\[w = \frac{19B}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.723 \text{ T})}{1.16 \times 10^{-26} \text{ kg}} = 9.97 \times 10^6 \text{ rad/s}
\]
7. In the circuit shown below both batteries have insignificant internal resistance.  
\[ \varepsilon = 20.0\text{ V} \]
\[ 30.0\ \Omega \]
\[ 75.0\ \Omega \]
\[ 20.0\ \Omega \]
\[ 25.0\ \text{V} \]
\[ 50.0\ \Omega \]

(a) What will the current in the ammeter be before the switch S is closed?

\[ I = \frac{\varepsilon}{R} = \frac{20.0}{50.0} = 0.400\ \text{A} \]

(b) What will the current in the ammeter be after the switch S is closed?

\[ I_A = \frac{\varepsilon}{R} = \frac{25.0}{50.0} = 0.500\ \text{A} \]

8. A closely wound, circular coil with radius 2.40 cm has 800 turns of wire and produces a magnetic field of 0.0580 T at the center of the coil.

(a) What value of current is required to produce this magnetic field?

\[ B = \mu_0 n I \]
\[ \Rightarrow \quad I = \frac{2BA}{\mu_0 n} = \frac{2 \times 0.0580 \times 2.40 \times 10^{-2}}{4 \times 10^{-7} \times 800} = 2.77\ \text{A} \]

(b) At what points along the axis is the magnetic field equal to half its value at the center. Take the center to be \( x = 0 \).

\[ \frac{\mu_0 n I a}{2} \left( x^2 + a^2 \right)^{3/2} = \frac{\mu_0 N I}{2} \]
\[ a \left( x^2 + a^2 \right)^{3/2} = \frac{\mu_0 N I}{2} \]

\[ 2a \left( x^2 + a^2 \right)^{3/2} = \mu_0 n I \]

\[ x^2 = a^2 \left( 4^{3/2} - 1 \right) = 0.587a^2 \]

\[ x = \pm 0.766a = \pm 1.84\ \text{cm} \]

I meant to remove part b, but didn't. Therefore I'll give 10 points for the successful completion of either part.
1. \( 20V - (I_1 + I_2) 20 \Omega = 0 \) 
\( I_1 = \frac{16}{15} I_2 \) 
\( I_2 = \frac{15}{16} I_1 \) 
\( I_2 = 1.5 \text{mA} \) 
\( I_1 = 1.65 \text{mA} \)

2. \( 20V - (I_1 + I_2) 20 \Omega = 0 \) 
\( I_1 = \frac{16}{15} I_2 \) 
\( I_2 = \frac{15}{16} I_1 \) 
\( I_2 = 1.5 \text{mA} \) 
\( I_1 = 1.65 \text{mA} \)

Total current: \( I_1 + I_2 = 341 \text{mA} \)
9. A circular loop with radius $r=0.0240$ m and resistance $R=0.610$ Ω is in a region of spatially uniform magnetic field, as shown in the figure on the right. The magnetic field is directed out of the page. Initially $B=6.00$ T and is increasing at a rate of $\frac{dB}{dt}=+0.340 T/s$

a. Calculate the induced voltage in the loop

\[ \mathcal{E} = -\frac{d\Phi}{dt} = -\pi r^2 \frac{dB}{dt} \]
\[ = -\pi (0.0240 m)^2 \left( \frac{340 T}{s} \right) = -0.15 \times 10^{-4} \text{ volts} \]

b. Show the direction of the induced current in the figure.

The current opposes the increase of $B$; clockwise.

10. In the figure on the right, switch $S_1$ is closed while switch $S_2$ is kept open. The inductance is $L=0.115$ H and the resistance is $120\ \Omega$.
When the current has reached its final value, the energy stored in the inductor is $0.260\ \text{J}$.

a. What is the emf $\mathcal{E}$ of the battery?

\[ U = \frac{1}{2} L i_0^2 = 0.260 \ \text{J} \Rightarrow i_0 = \sqrt{\frac{2U}{L}} \]
\[ i_0 = \sqrt{\frac{2(0.260)}{0.115}} = 2.13\ \text{A} \]
\[ \mathcal{E} = i_0 R = (2.13\ \text{A})(120\ \Omega) = 255\ \text{V} \]

b. After the current has reached its final value, switch $S_1$ is opened and switch $S_2$ is closed. How much time does it take for the energy in the inductor to decrease to half its original value?

\[ U = \frac{1}{2} L i^2 \text{ to reduce } U \Rightarrow U = \frac{U}{2} \text{ the } i \text{ must decrease to } \frac{\sqrt{2}}{2} i_0 \]
\[ i = i_0 e^{-\frac{R}{L}t} \]
\[ \frac{\sqrt{2}}{2} = e^{-\frac{R}{L}t} \Rightarrow t = -\frac{\ln\left(\frac{\sqrt{2}}{2}\right)}{R} = \frac{-\ln\left(\frac{\sqrt{2}}{2}\right)(0.115\ \text{H})}{120\ \Omega} \]
\[ t = 3.32 \times 10^{-4}\ \text{s} \]

11. When they are bored, Amateur Radio Operators bounce radio signals off the Moon. When they use a frequency of 144.1 MHz, on average, how long will it take for the signal to travel to the Moon and return to Earth?

\[ d = ct \Rightarrow \]
\[ t = \frac{d}{c} = \frac{2(3.84 \times 10^8 \text{ m})}{3.00 \times 10^8 \text{ m/s}} = 2.56 \text{ sec} \]
12. An L-R-C series circuit with \( L = 0.120 \, \text{H}, \, R = 240 \, \Omega \) and \( C = 7.30 \, \mu\text{F} \) carries an rms current of 0.450 A when the frequency is 400 Hz.

a. What is the impedance of this circuit?

\[
Z = \sqrt{R^2 + \left( \frac{2\pi f L - \frac{1}{2\pi f C}}{2\pi f C} \right)^2}
\]

\[
= \sqrt{(240 \, \Omega)^2 + \left( \frac{2\pi (400 \, \text{Hz})(0.120 \, \text{H}) - \frac{1}{2\pi (400 \, \text{Hz})(7.3 \times 10^{-6} \, \text{F})}}{2\pi (400 \, \text{Hz})(7.3 \times 10^{-6} \, \text{F})} \right)^2}
\]

\[
= 342 \, \Omega
\]

b. The frequency is lowered to 170 Hz, what is the new impedance of the circuit at the lower frequency?

\[
Z = \sqrt{(240 \, \Omega)^2 + \left( \frac{2\pi (170 \, \text{Hz})(0.120 \, \text{H}) - \frac{1}{2\pi (170 \, \text{Hz})(7.3 \times 10^{-6} \, \text{F})}}{2\pi (170 \, \text{Hz})(7.3 \times 10^{-6} \, \text{F})} \right)^2}
\]

\[
= 128 \, \Omega
\]