HYPOTHESIS TESTS FOR ONE POPULATION MEAN WORKSHEET
MTH 1210, SPRING 2019

We are responsible for 2 types of hypothesis tests that produce inferences about the unknown population mean, μ, each of which has 3 possible alternative hypothesis (6 cases total). As is the case with confidence intervals, our inferences are made with a prescribed certainty called the confidence level. For confidence intervals, we describe this with our confidence level (e.g. 90%, 95% or 99%). When we do hypothesis tests, we indicate our probability of making an error, which is α. The probability α is called our *significance level*. **It is crucial that α is determined prior to the hypothesis testing procedure; in fact, we should really choose α before observing the data in the sample in order for our underlying assumptions to be satisfied.**

The significance level α should be stated at the start of any hypothesis test. For any hypothesis testing procedure we then state a *null hypothesis*, H_0. For the hypothesis testing procedure on this worksheet, H_0 is the hypothesis that the population mean, μ, is equal to a number. We often think about this number, denoted μ_0, as the presumed population mean. We use the hypothesis test to determine if there is strong evidence that this null hypothesis is *incorrect*.

The next step of the hypothesis test is our *alternative hypothesis*, H_a. This is a hypothesis that is contrary to our null hypothesis; if there is enough evidence for the alternative hypothesis, then the null hypothesis is rejected. The conclusion of our hypothesis tests are always either "reject the null hypothesis " or "fail to reject the null hypothesis."

As usual, the size of our sample is n, mean $\bar{x}$ and standard deviation is denoted s. Each of our 6 hypothesis tests uses a *test statistic* computed in terms of these sample statistics (and, possibly, the population standard deviation σ). We conclude our hypothesis test using one of the following two approaches:

Option I, Critical Value Approach: Using this approach, we identify a *rejection region* that is based on the z or t critical values from Table IV in our textbook Appendix A. If the test statistic is in that rejection region, we "reject the null hypothesis "; otherwise, we "fail to reject the null hypothesis." The number α represents the (small) probability that we reject the null hypothesis even though it is actually true.

Option II, P-value Approach: Using this approach, we identify a *P-value* that is calculated from our test z or t test statistics using Table II or from the "Detailed t-table Areas to the right of t," table passed out in class, respectively. If the P value is less than the number α, we "reject the null hypothesis "; otherwise, we "fail to reject the null hypothesis." The number α represents the (small) probability that we reject the null hypothesis even though it is actually true.

1. z-tests for the population mean (σ is known)

For these tests, we assume that σ is known. We also assume that n is relatively large (> 30) or that the underlying population is roughly normal.

1.1. **Two-sided z-test.** A two-sided test uses a double interval rejection region that includes both $-\infty$ and ∞.

- **Null Hypothesis** $H_0 : \mu = \mu_0$
- **Alternative Hypothesis** $H_A : \mu \neq \mu_0$
- **Test Statistic**
$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$
- **Critical Values** $-z_{\alpha/2}$ and $z_{\alpha/2}$
- **Rejection Region** $(-\infty, -z_{\alpha/2}]$ or $[z_{\alpha/2}, \infty)$
- P**-value** $2P[Z > |z|]$ where Z is a standard normal distribution.

1.2. **The Right-sided z-test.** For this test, we use a single interval rejection region.

- **Null Hypothesis** $H_0 : \mu = \mu_0$
- **Alternative Hypothesis** $H_A : \mu > \mu_0$
- **Test Statistic**
$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$
- **Critical Value** z_α
- **Rejection Region** $[z_\alpha, \infty)$
- P**-value** $P[Z > z]$ where Z is a standard normal distribution.

1.3. **The Left-sided z-test.** For this test, we assume that σ is known, and we use a single interval rejection region.

- **Null Hypothesis** $H_0 : \mu = \mu_0$
- **Alternative Hypothesis** $H_A : \mu < \mu_0$
- **Test Statistic**
$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$
- **Critical Value** $-z_\alpha$
- **Rejection Region** $(-\infty, -z_\alpha]$
- P**-value** $P[Z < z]$ where Z is a standard normal distribution.

2. t-tests for the population mean (σ is unknown)

For these tests, we assume that σ is unknown. We also assume that n is relatively large (> 30) or that the underlying population is roughly normal.

2.1. **The Two-sided t-test.** For this we use a double interval rejection region that includes both $-\infty$ and ∞.

- **Null Hypothesis** $H_0 : \mu = \mu_0$
- **Alternative Hypothesis** $H_A : \mu \neq \mu_0$
- **Test Statistic**
$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$
- **Critical Values** $-t_{\alpha/2,n-1}$ and $t_{\alpha/2,n-1}$
- **Rejection Region** $(-\infty, -t_{\alpha/2,n-1}]$ or $[t_{\alpha/2,n-1}, \infty)$

- **P-value** $2P[T > |t|]$ where T is t-distribution with $n-1$ degrees of freedom.

2.2. **The Right-sided t-test.** For this test, we use a single interval rejection region.

- **Null Hypothesis** $H_0 : \mu = \mu_0$
- **Alternative Hypothesis** $H_A : \mu > \mu_0$
- **Test Statistic**
$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$
- **Critical Value** $t_{\alpha,n-1}$
- **Rejection Region** $[t_{\alpha,n-1}, \infty)$
- **P-value** $P[T > t]$ where T is t-distribution with $n-1$ degrees of freedom.

2.3. **The Left-sided t-test.** For this test, we use a single interval rejection region.

- **Null Hypothesis** $H_0 : \mu = \mu_0$
- **Alternative Hypothesis** $H_A : \mu < \mu_0$
- **Test Statistic**
$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$
- **Critical Value** $-t_{\alpha,n-1}$
- **Rejection Region** $(-\infty, -t_{\alpha,n-1}]$
- **P-value** $P[T < t]$ where T is t-distribution with $n-1$ degrees of freedom.

3. Exercises

For these exercises, use the appropriate test based on the information given (σ, s or $\hat{p}$), alternative hypothesis specified and significance level, α. You can assume that the underlying population is normally distributed.

(1) $\bar{x} = 300, \sigma = 37, n = 19, H_0 : \mu = 320, H_A : \mu \neq 320,\ \alpha = .01$

(2) $\bar{x} = 300, \sigma = 37, n = 19, H_0 : \mu = 320, H_A : \mu < 320,\ \alpha = .01$

(3) $\bar{x} = 330, \sigma = 53, n = 49, H_0 : \mu = 320, H_A : \mu > 320,\ \alpha = .1$

(4) $\bar{x} = 327, s = 31, n = 24, H_0 : \mu = 320, H_A : \mu > 320,\ \alpha = .1$

(5) $\bar{x} = 327, s = 15, n = 24, H_0 : \mu = 320, H_A : \mu \neq 320,\ \alpha = .05$

(6) $\bar{x} = 305, s = 23, n = 10, H_0 : \mu = 320, H_A : \mu < 320,\ \alpha = .05$

(7) A simple random sample of 36 St Bernard dog weights yields a sample mean of 193 pounds. It is known that the standard deviation of the population of all St Bernard dog weights is 18 pounds. Test the null hypothesis that the mean weight of all St Bernard dogs is 200 pounds (use a left-sided test and significance level $\alpha = .01$). State the conclusion in terms of the problem and calculate the P-value for this test.

(8) A simple random sample of 21 chihuahua dog weights yields a sample mean of 5.6 pounds. It is known that the standard deviation of the population of all chihuahua weights is 1.8 pounds. Test the null hypothesis that the mean weight of all chihuahuas is 4.6 pounds at the $\alpha = .1$ significance level. Use a two-sided test. State the conclusion in terms of the problem and calculate the P-value for this test.

(9) Eleven regions in the Congolese rain forest are randomly sampled. In each region rainfall was monitored for one year, and the following total yearly rainfalls, in centimeters, were reported:

$$\{276, 255, 255, 297, 213, 241, 269, 262, 145, 185, 209\}$$

Assume that yearly rainfalls within the Congolese rain forest are distributed normally. Test the null hypothesis that the mean yearly rainfall of all locations in the Congolese rain forest is 200. Use a two-sided test with $\alpha = .01$. State the conclusion in terms of the problem.

(10) Twelve regions in an Amazon rain forest are randomly sampled. The total yearly rainfalls, in centimeters, for these regions were reported as follows:

$$\{188, 232, 210, 198, 202, 193, 219, 202, 252, 156, 184, 222\}$$

Assume that yearly rainfalls within the Amazon rain forest are distributed normally. Test the null hypothesis that the mean yearly rainfall of all locations in the Amazon rain forest is 200. Use a two-sided test with $\alpha = .05$. State the conclusion in terms of the problem.

4. Solutions to Exercises

(1) Solution:
- **Test Statistic**
$$z = \frac{300 - 320}{\frac{37}{\sqrt{19}}} = -2.36$$
- **Rejection Region** $(-\infty, -2.576]$ or $[2.576, \infty)$
- **Conclusion** Fail to reject the null hypothesis at the 1% significance level.

(2) Solution:
- **Test Statistic**
$$z = \frac{300 - 320}{\frac{37}{\sqrt{19}}} = -2.36$$
- **Rejection Region** $(-\infty, -2.33]$
- **Conclusion** Reject the null hypothesis at the 1% significance level.

(3) Solution:
- **Test Statistic**
$$z = \frac{330 - 320}{\frac{53}{\sqrt{49}}} = 1.32$$
- **Rejection Region** $[1.28, \infty)$
- **Conclusion** Reject the null hypothesis at the 10% significance level.

(4) Solution:
- **Test Statistic**
$$t = \frac{327 - 320}{\frac{31}{\sqrt{24}}} = 1.106$$
- **Rejection Region** $[1.32, \infty)$
- **Conclusion** Fail to reject the null hypothesis at the 10% significance level.

(5) Solution:
- **Test Statistic**
$$t = \frac{327 - 320}{\frac{15}{\sqrt{24}}} = 2.29$$
- **Rejection Region** $(-\infty, -2.07]$ or $[2.07, \infty)$
- **Conclusion** Reject the null hypothesis at the 5% significance level.

(6) Solution:
- **Test Statistic**
$$t = \frac{305 - 320}{\frac{23}{\sqrt{10}}} = -2.06$$
- **Rejection Region** $(-\infty, -1.83]$
- **Conclusion** Reject the null hypothesis at the 5% significance level.

(7) We have that that $n = 36$, $\overline{x} = 193$, and $\sigma = 18$. The critical value of z we need is $z_{.01} = 2.32$ since we are using a one-sided z-test with $\alpha = .01$. The hypothesis test is then carried out as follows:
- **Test Statistic**
$$z = \frac{193 - 200}{\frac{18}{\sqrt{36}}} = -2.33$$

- **Rejection Region** $(-\infty, -2.326]$
- **Conclusion** Reject the null hypothesis at the 1% significance level.
- **Conclusion in terms of the problem** Reject the null hypothesis that the mean weight of all St Bernards is 200 pounds at the 1% level.

(8) Solution:

We are given the population standard deviation, σ, and our alternative hypothesis is two-sided, so we use the two-sided z-test.

- **Test Statistic**
$$z = \frac{5.6 - 4.6}{\frac{1.8}{\sqrt{21}}} = 2.54$$
- **Critical Value** $\pm z_{.05} = \pm 1.64$
- **Rejection Region** $(-\infty, -1.64]$ or $[1.64, \infty)$
- **Conclusion** Reject the null hypothesis at the 10% significance level.
- **Conclusion in terms of the problem** Reject the null hypothesis that the true mean weight of all chihuahua dogs is 4.6 pounds at the 10% significance level.

(9) By inputing the data into our calculator, we find that $n = 11$, $\overline{x} = 237$, and $s = 44.68$. The critical value of t we need is $t_{.005,10} = 3.17$ since we are using a two-sided t-test with $\alpha = .01$ and $n - 1 = 10$. The hypothesis test is then carried out as follows:

- **Test Statistic**
$$t = \frac{237 - 200}{\frac{44.68}{\sqrt{11}}} = 2.74$$
- **Rejection Regions** $(-\infty, -3.17]$ and $[3.17, \infty)$
- **Conclusion** Fail to reject the null hypothesis at the 1% significance level.
- **Conclusion in terms of the problem** Fail to reject the null hypothesis that the mean yearly rainfall of locations within the Congolese rain forest is 200 centimeters per year at the 1% level.

(10) We are only given the sample standard deviation, s, and our alternative hypothesis is two-sided, so we use the two-sided t-test. We find that the mean and standard deviation of this sample are given by $\overline{x} = 204.8$ and $s = 24.8$, respectively.

- **Test Statistic**
$$t = \frac{204.8 - 200}{\frac{24.8}{\sqrt{12}}} = .67$$
- **Critical Value** $\pm t_{.005,11} = \pm 2.201$
- **Rejection Region** $(-\infty, -2.201]$ or $[2.201, \infty)$
- **Conclusion** Fail to reject the null hypothesis at the 5% significance level.
- **Conclusion in terms of the problem** Fail to reject the null hypothesis that the true mean yearly rainfall at locations in the Amazon rain forest is 200 inches per year at the 5% significance level.