10.2 Sum and Difference Identities

True or False? can we distribute "cos" through parentheses?

ie is $\cos(A-B) = \cos(A) - \cos(B)$ always true?

Try for $A = 45°$ and $B = 45°$

$$\cos(A-B) = \cos(45° - 45°) = \cos(0°) = 1$$

$$\cos(A) - \cos(B) = \cos(45°) - \cos(45°) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$$

so $\cos(A-B) \neq \cos(A) - \cos(B)$!

What is $\cos(A-B)$ equal to?

Answer - You need a huge leap of faith here - or watch the video ☆

$$\boxed{\cos(A-B) = \cos A \cos B + \sin A \sin B}$$ ← Difference Identity for Cosine

Why do we care?

(1) Emphasize that trig functions have different properties than, say, numbers

see also natural log function

(2) be familiar with these identities for future Calc classes

(3) Allow us to calculate more trig values by hand

Ex Find $\cos(15°)$ without a calculator

Our special angles are $30°, 45°, 60°$ and larger angles with these reference angles

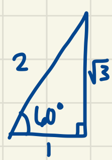
$$\underset{A}{15°} = \underset{A}{60°} - \underset{B}{45°}$$

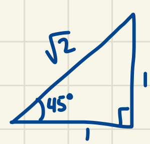
$$\cos(15°) = \cos(\overset{A}{60°} - \overset{B}{45°})$$

$$= \cos 60° \cdot \cos 45° + \sin 60° \sin 45°$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$





What about $\cos(A+B)$?

sum of 2 angles

How to derive formula for $\cos(A+B)$ from $\cos(A-B)$

$$\cos(A+B) = \cos(A-(-B))$$
$$= \cos A \underbrace{\cos(-B)}_{\cos(-B)=\cos(B)} + \sin A \underbrace{\sin(-B)}_{\sin(-B)=-\sin(B)}$$
$$= \cos A \cos B + \sin A(-\sin B)$$
$$= \cos A \cos B - \sin A \sin B$$

We know

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\boxed{\cos(A+B) = \cos A \cos B - \sin A \sin B}$$

Sum and Difference Identities for Sine and Cosine

$\cos(A-B) = \cos A \cos B + \sin A \sin B$
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\sin(A-B) = \sin A \cos B - \cos A \sin B$

Ex Find $\sin\left(\frac{5\pi}{12}\right)$ (no calculator)

$$\frac{5\cancel{\pi}\,\cancel{\text{rad}}}{\cancel{12}_{\,1\ 2}} \times \frac{\cancel{180}^{\,30\ 15}\text{deg}}{\cancel{\pi}\,\cancel{\text{rad}}} = 75°$$

Ask can I add or subtract two special angles to get 75° ?

Many ways

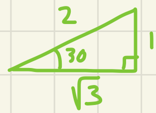
$75° = 30° + 45°$

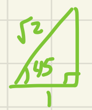
$75° = 120° - 45°$

$75° = 135° - 60°$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\sin\left(\frac{5\pi}{12}\right) = \sin(75°)$$
$$= \sin(\underset{A}{30°} + \underset{B}{45°})$$
$$= \sin 30° \cos 45° + \cos 30° \sin 45°$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$
$$= \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$





Try using $75° = 120° - 45°$ or $75° = 135° - 60°$