# Probability and Statistics

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January 22, 2019

# Topics

# Objectives

Objectives:

- Identify sources of variation in data
- Recognize the four types of variables
- Construct a frequency distribution (table) and histogram

# Introduction (1.1)

- ***Statistics*** is the science of collecting, organizing, analyzing, and drawing conclusions from data.
  - Understanding variability in data
  - Distinguishing signal from noise
  - Assessing the evidence data provides
  - Using data to answer scientific questions

# Introduction (1.1)

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  - Understanding variability in data
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  - Assessing the evidence data provides
  - Using data to answer scientific questions
- Variation in data arises from many sources.

## Example

In a study to assess the effect of exercise on cholesterol levels, one group is assigned to an exercise regimen and other isn't. Is cholesterol reduced in exercise group?

Some **sources of variability**:

- People have naturally different cholesterol levels
- Respond differently to same amount of exercise (e.g. genetics)
- May vary in adherence to exercise regimen
- Diet may have an effect

## Example

Surface water specimens taken upstream and downstream of landfill are analyzed for certain toxins. Can these be used to show toxins are leaching from landfill?

Some **sources of variability**:

- Natural variation; where specimens were taken
- Specimens gathered under different conditions
- Lab error or variability in analyzing specimens

- Statistics fits into a broader **scientific process**:
  - Scientific question.
  - Design experiment / study.
  - Perform experiment / collect data.
  - Summarize and analyze data.
  - Draw statistical conclusions
  - Draw scientific conclusions.

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- A ***sample*** is a subset of the population, usually selected randomly.

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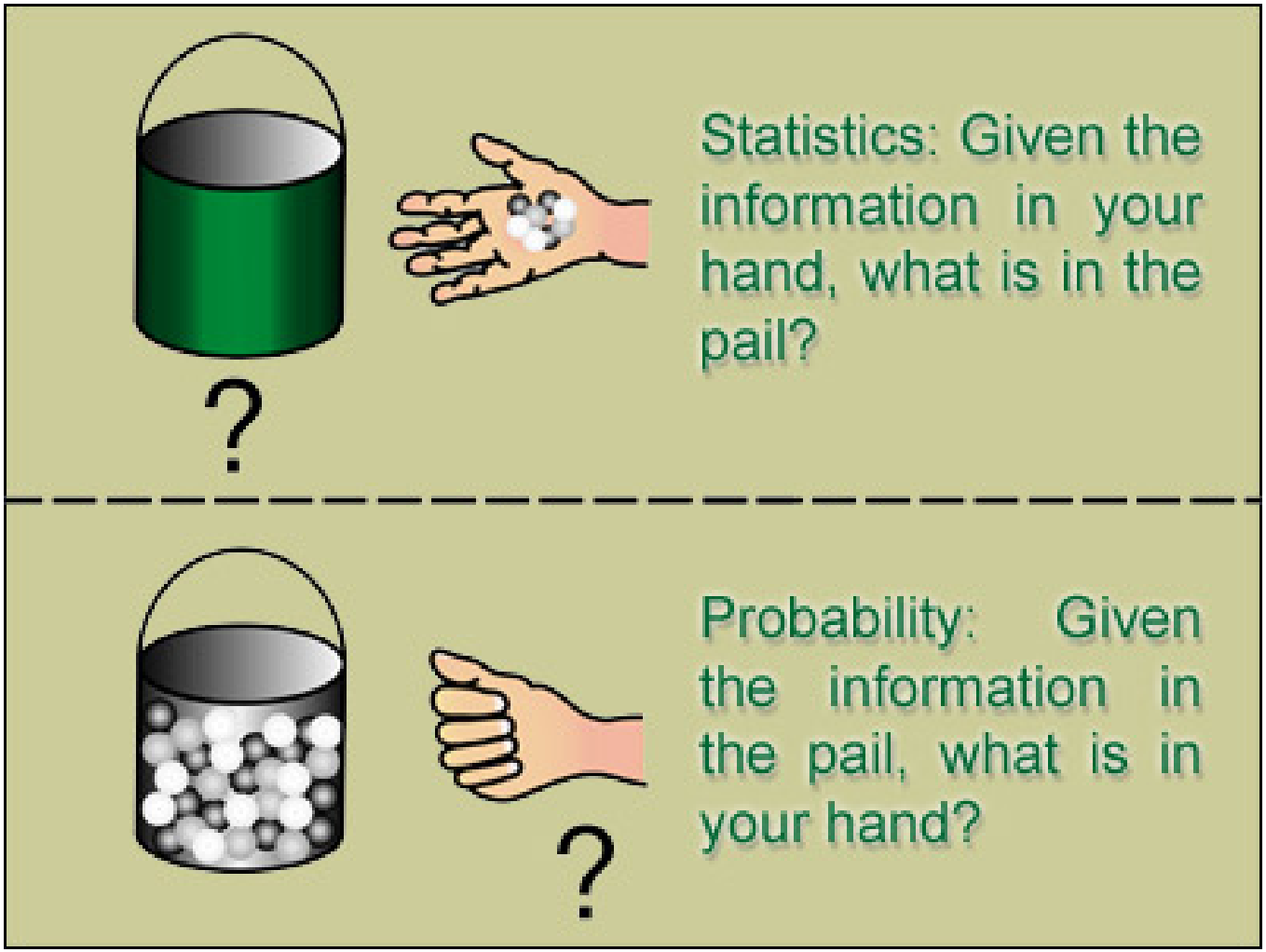
  Need to learn the language of probability in order to communicate statistical conclusions.


Statistics: Given the information in your hand, what is in the pail?
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# Variables and Data (1.2)

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- Variables can be ***categorical*** (or ***qualitative***) (taking values in a set of categories) or ***numerical*** (or ***quantitative***) (taking numerical values).

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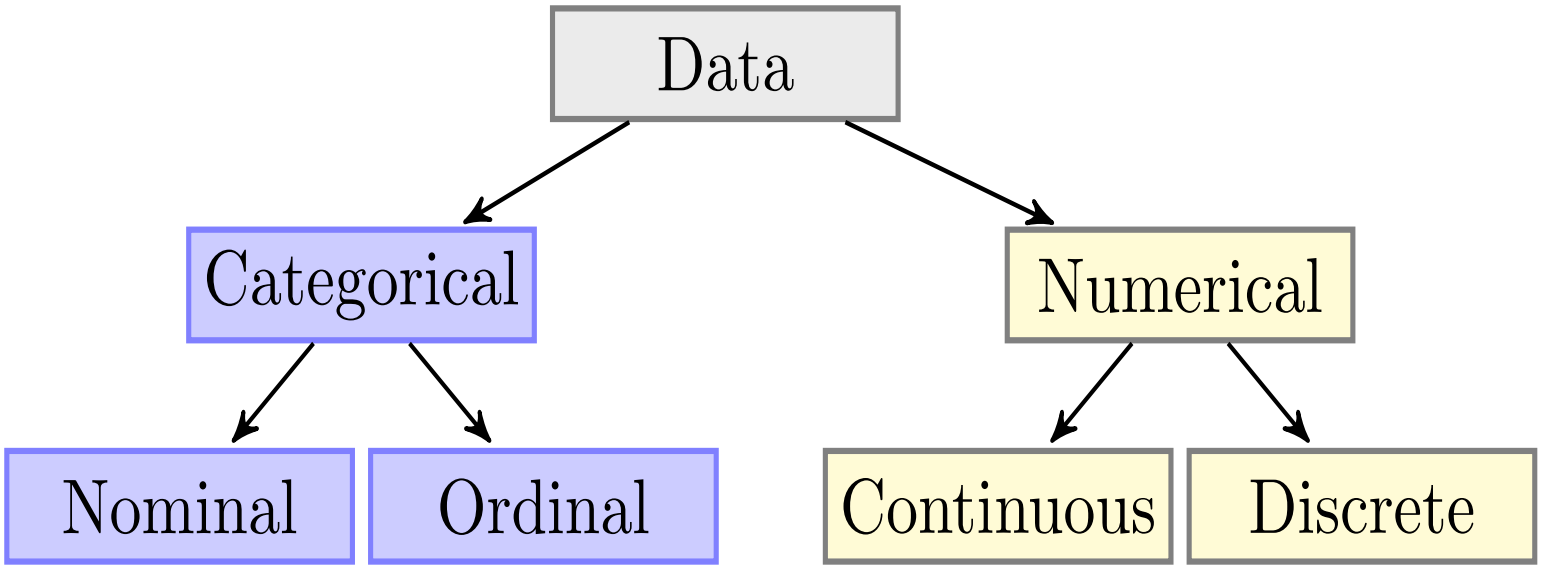
- Numerical variables can be:
  - ***Discrete*** (the set of possible values is finite or countably infinite, i.e. they're isolated numbers with gaps between them, e.g. integers)
  - ***Continuous*** (the set of possible values is a continuum, or interval).


Data
Categorical
Numerical
Nominal
Ordinal
Continuous
Discrete

# Frequency Distributions and Histograms (1.2)

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  Displays the number (***frequency***) and/or proportion (***relative frequency***) of times each value of a variable $x$ occurs.

## Example

Size of litters for $n = 36$ sows. Litter size (number of piglets) is an integer (discrete). The data:

| | | | | | |
|---|---|---|---|---|---|
| 10 | 12 | 10 | 7 | 14 | 11 |
| 14 | 11 | 10 | 13 | 10 | 10 |
| 8 | 11 | 7 | 13 | 12 | 13 |
| 10 | 8 | 5 | 11 | 11 | 12 |
| 11 | 11 | 9 | 8 | 12 | 10 |
| 9 | 11 | 10 | 12 | 10 | 9 |

Here's the **frequency distribution** (table):

| Litter Size ($x$) | Number of Sows (Frequency) | Relative Frequency |
|---|---|---|
| 5 | 1 | 0.028 |
| 6 | 0 | 0.000 |
| 7 | 2 | 0.056 |
| 8 | 3 | 0.083 |
| 9 | 3 | 0.083 |
| 10 | 9 | 0.250 |
| 11 | 8 | 0.222 |
| 12 | 5 | 0.139 |
| 13 | 3 | 0.083 |
| 14 | 2 | 0.056 |

- ***Histogram for Discrete Data***: A graph of the frequency distribution.

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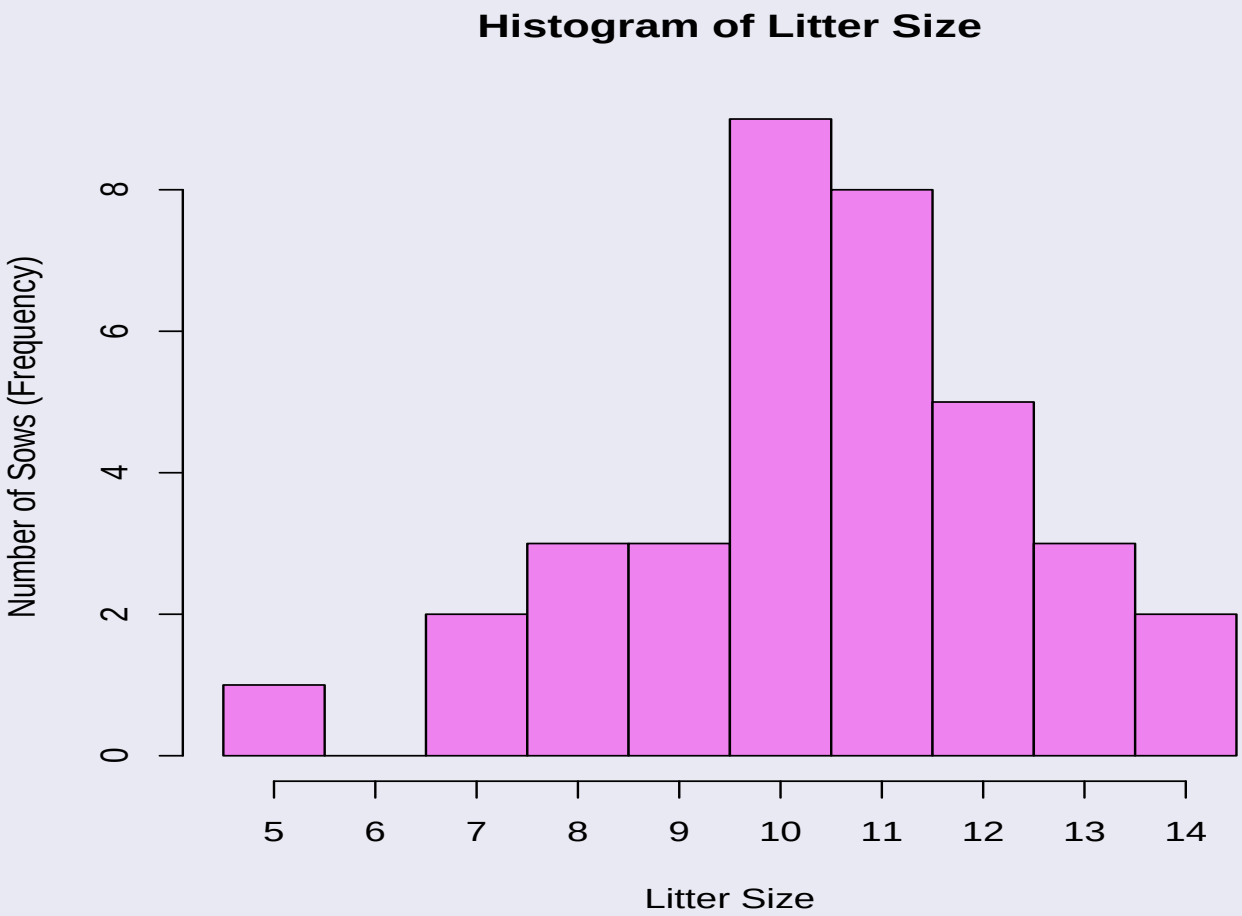
  - Mark the possible values $x$ on the horizontal axis.
  - Above each value, draw a rectangle whose height is the frequency (or relative frequency) of that value.

# Example (Cont'd)



- ***Frequency Distribution for Continuous Data***: A way to summarize the data.

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  - Divide the measurement axis of $x$ into a suitable number of ***class intervals***.
  - Displays the number (***frequency***) and/or proportion (***relative frequency***) of data observations that fall into each interval.
  - Use right-open, left-closed class intervals (i.e. $a \leq x < b$) so that a data observation falling on a boundary goes in interval to the *right* of the boundary.

## Example

Here are surgery times (in hours) for emergency surgeries of $n = 50$ animals at a local animal hospital (ordered from shortest to longest).

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0.33 | 0.33 | 0.33 | 0.33 | 0.50 | 0.50 | 0.50 | 0.67 | 0.75 |
| 0.75 | 0.75 | 0.83 | 0.92 | 0.92 | 1.00 | 1.00 | 1.00 | 1.00 |
| 1.08 | 1.08 | 1.17 | 1.25 | 1.27 | 1.33 | 1.42 | 1.42 | 1.50 |
| 1.50 | 1.50 | 1.50 | 1.58 | 1.67 | 1.67 | 1.67 | 1.67 | 1.67 |
| 1.70 | 1.75 | 1.75 | 1.83 | 1.83 | 2.00 | 2.00 | 2.00 | 2.33 |
| 2.42 | 2.50 | 2.67 | 3.08 | 4.50 | | | | |

Here's the **frequency distribution** (table):

| Class Interval | Number of Surgeries (Frequency) | Relative Frequency |
|---|---|---|
| 0.0 - $<$ 0.5 | 4 | 0.08 |
| 0.5 - $<$ 1.0 | 10 | 0.20 |
| 1.0 - $<$ 1.5 | 12 | 0.24 |
| 1.5 - $<$ 2.0 | 15 | 0.30 |
| 2.0 - $<$ 2.5 | 5 | 0.10 |
| 2.5 - $<$ 3.0 | 2 | 0.04 |
| 3.0 - $<$ 3.5 | 1 | 0.02 |
| 3.5 - $<$ 4.0 | 0 | 0.00 |
| 4.0 - $<$ 4.5 | 0 | 0.00 |
| 4.5 - $<$ 5.0 | 1 | 0.02 |

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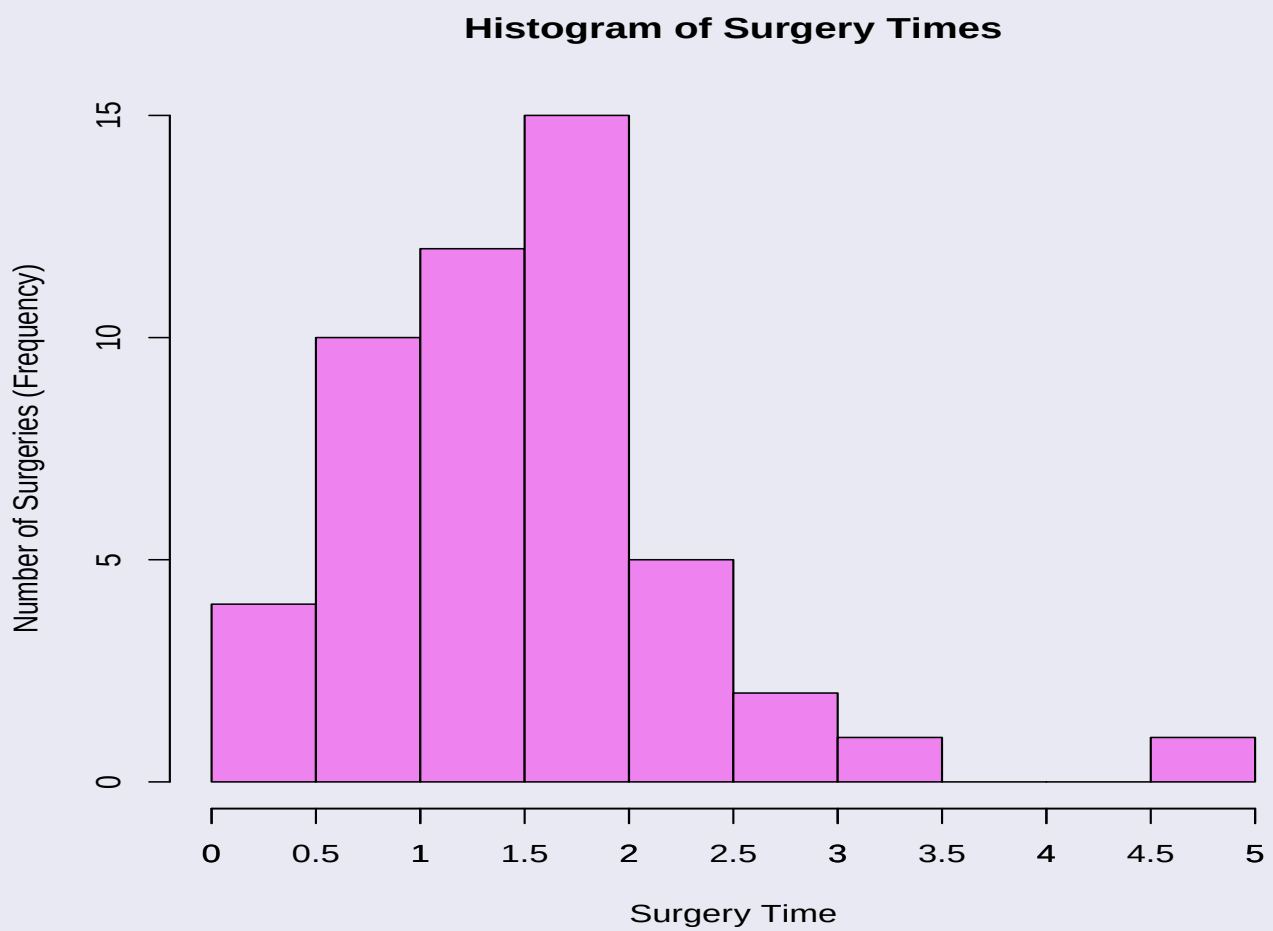
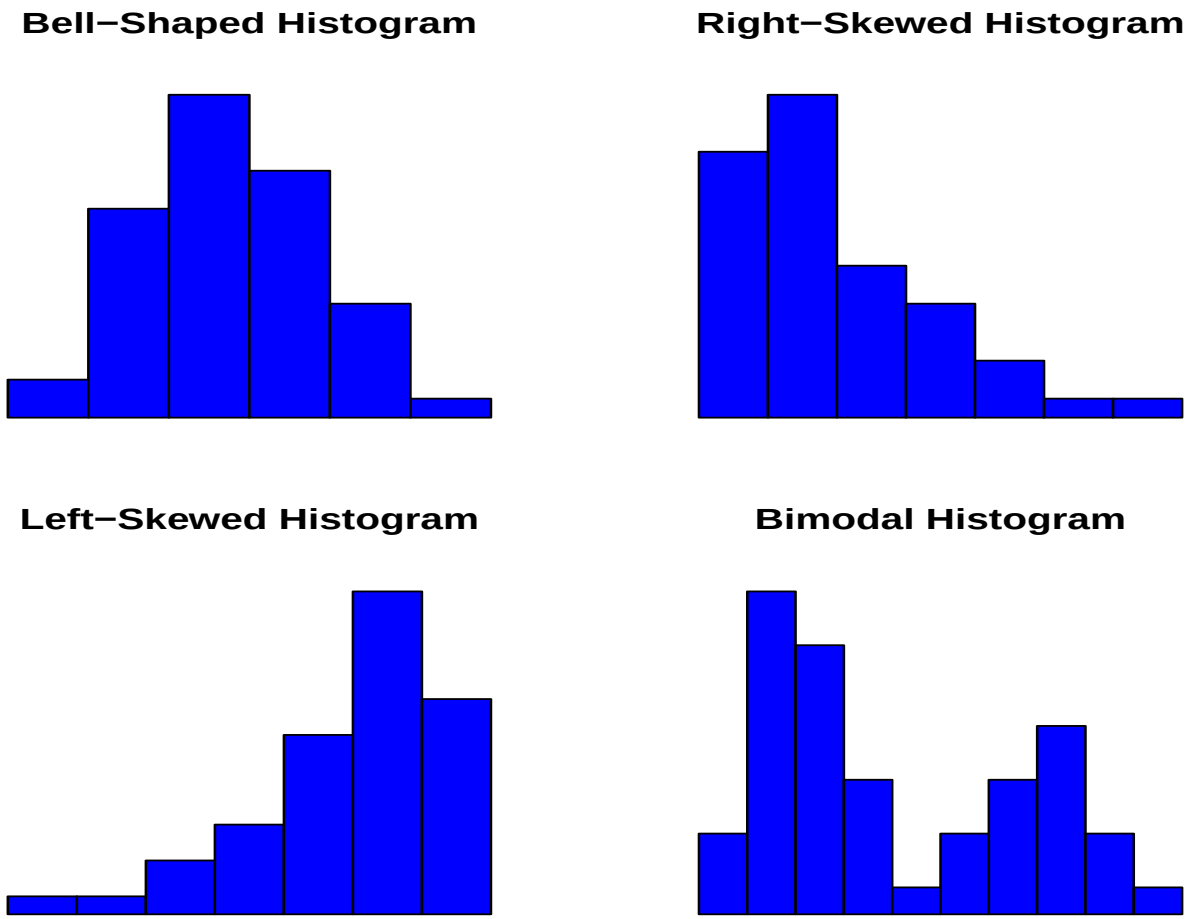
  - Mark the class interval endpoints on the horizontal $x$ axis.
  - Above each class interval, draw a rectangle whose height is the frequency (or relative frequency) of that interval.

# Example (Cont'd)



The figure below illustrates some common **histogram shapes**.



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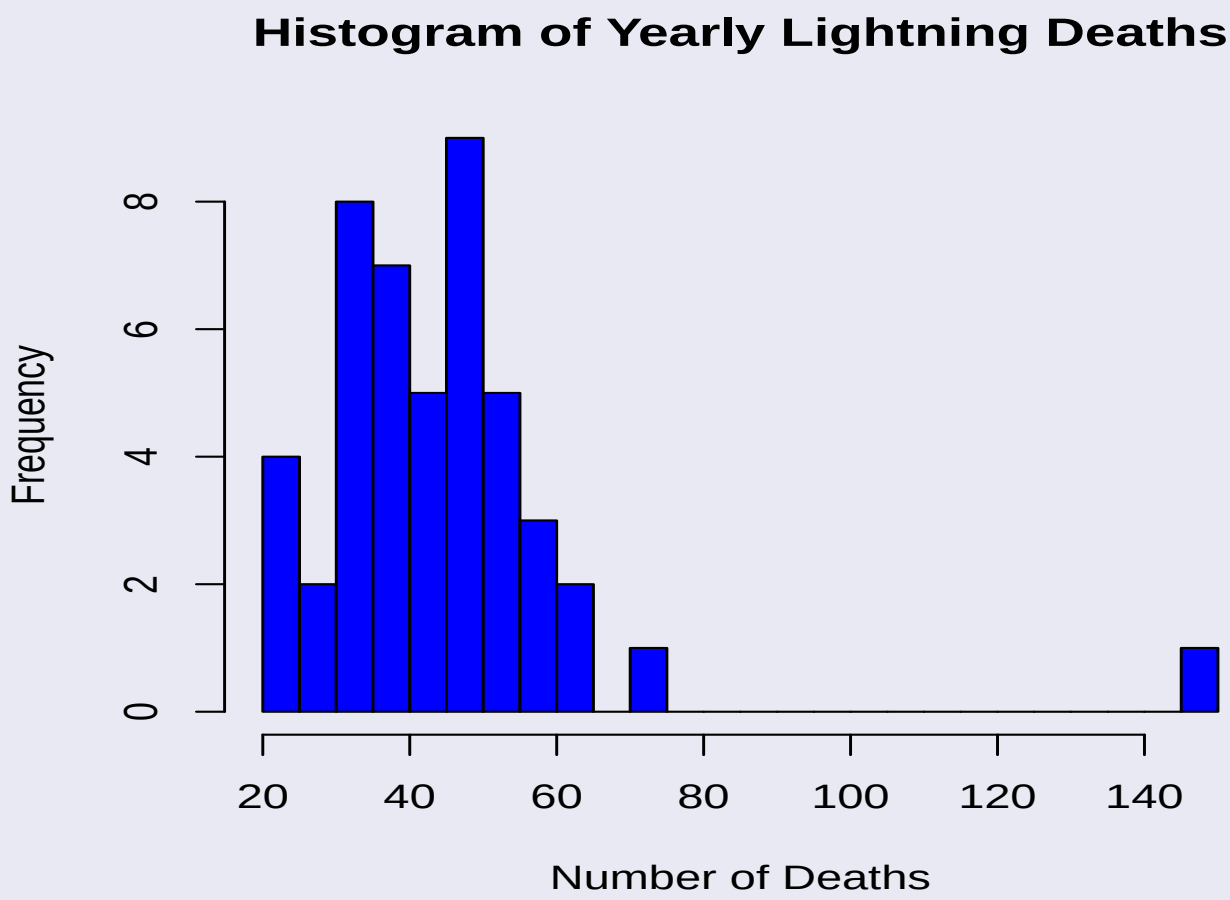
  - ***Bimodal*** – two distinct ”mounds”
  - ***Multimodal*** – multiple ”mounds”
- Histograms can also reveal ***outliers*** (values far away from the rest of the data).

## Example

Here's (part of) a data set on the numbers deaths by lightning strikes in the U.S. for each of the years 1959 - 2005, as compiled from reports by the National Weather Service.

| Year | Deaths |
|---|---|
| 1959 | 75 |
| 1960 | 48 |
| 1961 | 61 |
| 1962 | 48 |
| 1963 | 150 |
| 1964 | 49 |
| ⋮ | ⋮ |
| 2005 | 38 |


Histogram of Yearly Lightning Deaths
Frequency
0
2
4
6
8
20
40
60
80
100
120
140
Number of Deaths

Regarding the **outlier**, the National Weather Service report states:

> *On December 8, 1963 the crash of a jetliner killing 81 people near Elkin, Maryland, was attributed to lightning by the Civil Aeronautics Board investigators.*

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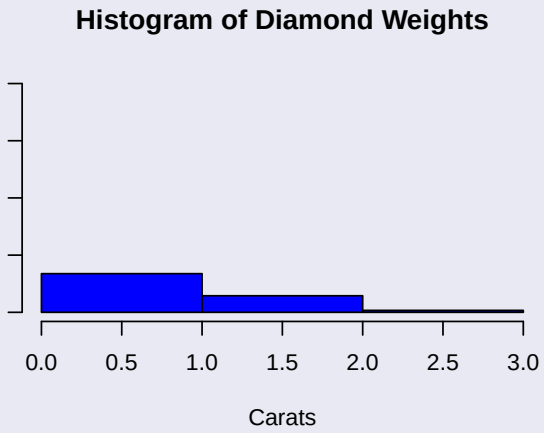
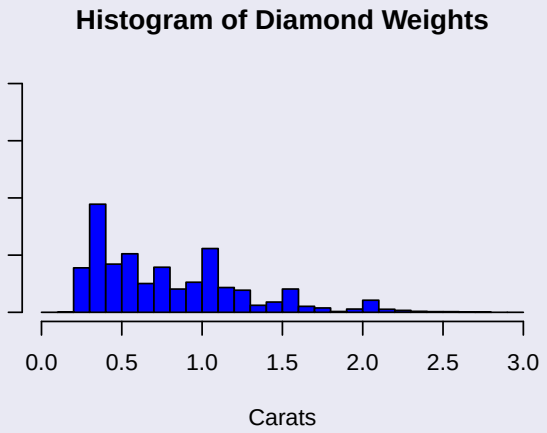
  - Each data value must go in exactly one class (i.e. no overlapping classes and no gaps between neighboring classes)
  - Classes should (usually) be all the same width
  - Use sensible and convenient boundaries, e.g. 20-30 not 19.83-28.87.
  - Usually 5-15 intervals works well. For larger data sets, more than 15 might be better.
    - Using too few can hide important details in the data.
    - Using too many intervals can show too much detail.

## Example

Here's (part of) a data set containing the weights in carats of $n = 50,000$ cut diamonds from reputable dealers around the world. A carat is 0.2 of a gram.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0.36 | 1.20 | 0.72 | 0.54 | 0.33 | 0.71 | 1.01 | 1.20 | 2.22 |
| 0.90 | 0.55 | 0.77 | 1.21 | 1.22 | 2.01 | 0.31 | 0.32 | 0.73 |
| 0.32 | 1.47 | 2.00 | 0.33 | 1.01 | 1.50 | 0.72 | 0.33 | 0.27 |
| | | | | ⋮ | | | | |
| 0.44 | 0.50 | 0.31 | 0.34 | 0.90 | 0.70 | 1.52 | 1.10 | 0.76 |


Histogram of Diamond Weights
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Carats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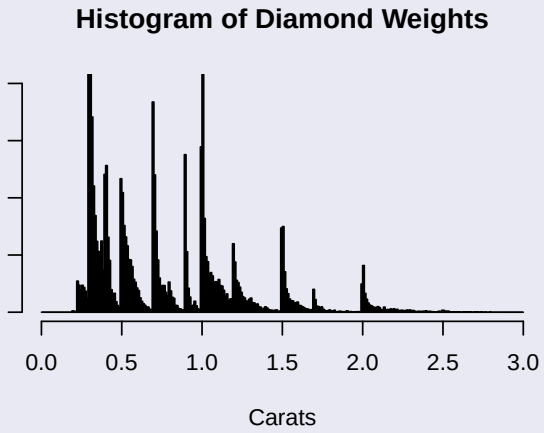
Carats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0.0
0.5
1.0
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