Probability and Statistics

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Topics

Objectives

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- Obtain percentiles from the cumulative distribution function of a continuous random variable.

Cumulative Distribution Functions (Cont'd) (4.2)

Properties of Cumulative Distribution Functions:

1. $F(x)$ is non-decreasing.
2. $\lim_{x\to\infty} F(x) = 1$.
3. $\lim_{x\to-\infty} F(x) = 0$.
4. If X is *continuous* with pdf $f(x)$, then $F(x)$ is also continuous.
5. If X is *discrete* with pmf $p(x)$, then $F(x)$ is a (right-continuous) step function, with steps of size $p(x)$ at each of the possible values x of X.

Percentiles of Continuous Distributions (4.2)

- For $0 < p < 1$, the $\mathbf{100}p$***th percentile*** of the distribution of a **continuous** random variable X is the value η defined by

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Examples:

- The **90th percentile** of scores on the verbal SAT is $\eta = 600$, and is the score below which **90%** of all scores lie.
- The **99th percentile** of U.S. incomes is $\eta = \$400,000$ (according to a 2012 CNN news report), and is the income below which **99%** of all incomes lie.

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Finding Percentiles: The $100p$**th percentile** of the distribution of a continuous random variable X whose **cdf** is $F(x)$ is obtained by solving

$$F(\eta) \;=\; p$$

for η.

Example

Let

$$X = \text{A company's profit (in millions of dollars) in the coming year}$$

Suppose the **pdf** of X is

$$f(x) = \begin{cases} \frac{3}{x^4} & \text{for } x \geq 1 \\ 0 & \text{for } x < 1 \end{cases}$$

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so the **cdf** of X is

$$F(x) = \begin{cases} 1 - \frac{1}{x^3} & \text{for } x \geq 1 \\ 0 & \text{for } x < 1 \end{cases}$$

The **75th percentile** of the distribution of X is the value marked η in the graphs on the next slide.

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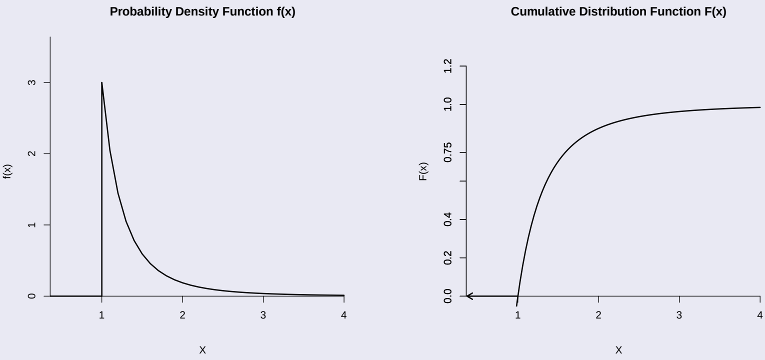
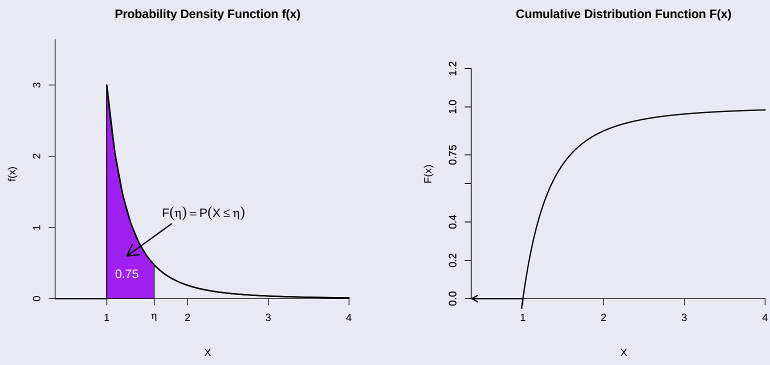
η is the profit level that will be exceeded with probability 0.25.


Probability Density Function f(x)
f(x)
0
1
2
3
1
2
3
4
X
Cumulative Distribution Function F(x)
F(x)
0.0
0.2
0.4
0.75
1.0
1.2
1
2
3
4
X


Probability Density Function f(x)
f(x)
F(η) = P(X ≤ η)
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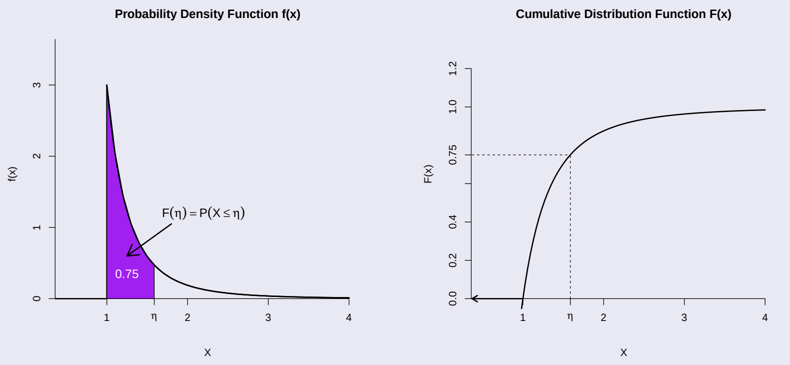
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