Probability and Statistics

Nels Grevstad

Metropolitan State University of Denver

ngrevsta@msudenver.edu

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Topics

Objectives

Objectives:

- Recognize exponential random variables.
- Use the exponential distribution to find probabilities.
- Find percentiles of the exponential distribution.
- State the relationship between a Poisson process and exponential random variables.
- Use the memoryless property to find exponential probabilities.

Exponential Random Variables (4.4)

- **Exponential random variables** are used to model **waiting times** for events that occur at random time points.

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  - The waiting time for the next customer to arrive at a store's checkout counter.

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  - The waiting time for a meteor ("shooting star") to appear in the night sky.
  - The waiting time for the next automobile to arrive at an intersection.
  - The waiting time for the next customer to arrive at a store's checkout counter.

- We'll see that the **memoryless property** makes exponential random variables suitable for modeling waiting times.

- The ***exponential distribution*** with **parameter** λ has **pdf**

Exponential(λ) **Pdf**:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

where $\lambda > 0$.

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$$\boldsymbol{X} \sim \textbf{exponential}(\boldsymbol{\lambda})$$

when X follows an exponential distribution.

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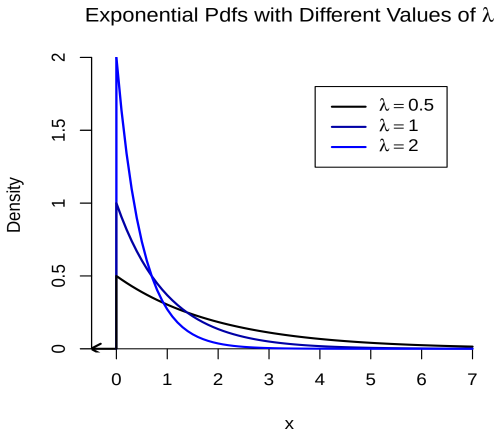
Exponential Pdfs with Different Values of λ
Density
x
λ = 0.5
λ = 1
λ = 2
0
0.5
1
1.5
2
0 1 2 3 4 5 6 7

- The mean and variance of an exponential random variable are:

Exponential Mean and Variance: If $X \sim \text{exponential}(\lambda)$ then

$$\begin{aligned} E(X) &= \frac{1}{\lambda} \\ V(X) &= \frac{1}{\lambda^2} \end{aligned}$$

Proofs: To show that $E(X) = 1/\lambda$, recall that *integration by parts* says:

$$\int u\,dv\,dx \;=\; uv \;-\; \int v\,du\,dx.$$

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$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x\,f(x)\,dx \\ &= \int_0^{\infty} x\,\lambda e^{-\lambda x}\,dx \\ &= x\left(-e^{-\lambda x}\right)\Big|_0^{\infty} \;-\; \int_0^{\infty} -e^{-\lambda x}\,dx \end{aligned}$$

$$= \quad 0 - 0 \;-\; \left(\frac{1}{\lambda} e^{-\lambda x}\, dx\right)\Big|_0^\infty$$

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\begin{aligned}
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To show that $V(X) = 1/\lambda^2$, recall that

$$V(X) \; = \; E(X^2) \; - \; \mu^2,$$

where $\mu = E(X) = 1/\lambda$.

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so that

$$du \; = \; 2x \qquad \text{and} \qquad v \; = \; -e^{-\lambda x} \, .$$

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Now use integration by parts *again* on the integral above, to get

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from which it follows that

$$V(X) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}\,.$$

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  - $\boldsymbol{\lambda} = 1/E(X)$ is the **rate** (number of events per unit of time) .

- The **cdf** of an exponential random variable is:

Exponential(λ) Cdf:

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - e^{-\lambda x} & \text{for } x \geq 0 \end{cases}$$

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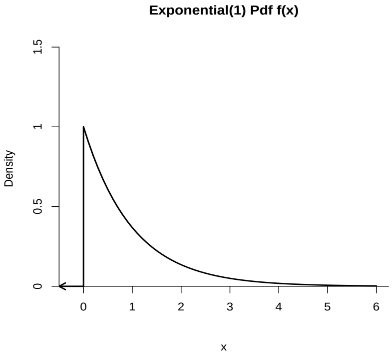
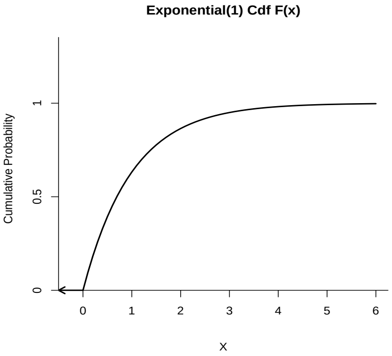

Exponential(1) Pdf f(x)
Density
0
0.5
1
1.5
0
1
2
3
4
5
6
x


Exponential(1) Cdf F(x)
Cumulative Probability
0
0.5
1
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3
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5
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X

Example

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To find the **probability** that you'll have to wait **longer than ten minutes**, To find the **probability** that you'll have to wait **longer than ten minutes**, either integrate the **pdf**:

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or just use the **cdf**:

$$\begin{aligned} P(X > 10) &= 1 - F(10) \\ &= 1 - (1 - e^{-0.1(10)}) \\ &= e^{-1} \\ &= \mathbf{0.3679}. \end{aligned}$$

To find the **probability** that you'll have to wait **between five and seven minutes**, either integrate the **pdf**:

$$P(5 < X \leq 7) \; = \; \int_5^7 \lambda e^{-\lambda x} \, dx$$

or just use the **cdf**:

$$\begin{aligned} P(5 < X \leq 7) &= F(7) \; - \; F(5) \\ &= (1 - e^{-0.1(7)}) \; - \; (1 - e^{-0.1(5)}) \\ &= e^{-0.5} \; - \; e^{-0.7} \\ &= \mathbf{0.1099}. \end{aligned}$$

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for η. This gives

$$\eta \;=\; -\frac{\log(0.5)}{0.1} \;=\; \mathbf{6.93} \quad \text{minutes.}$$

Relationship to the Poisson Process (4.4)

- **Exponential** random variables are related to the **Poisson process.**

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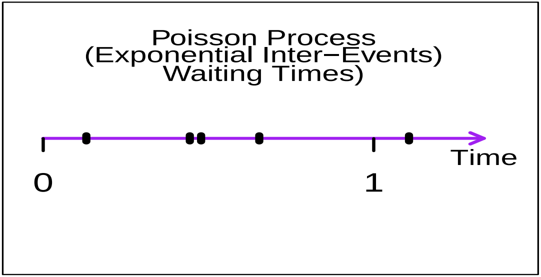
- **Exponential** random variables are related to the **Poisson process**.

Suppose the number of events occurring in any time interval of length t is a **Poisson** random variable with mean $\mu = \alpha t$ (where α, the **rate**, is the expected number of events in one unit of time), and that the numbers of events in non-overlapping time intervals are independent of each other.

Then the elapsed time between any two successive events is an **exponential**(λ) random variable with $\lambda = \alpha$.


Poisson Process
(Exponential Inter−Events)
Waiting Times)
0
1
Time

Memoryless Property (4.4)

- A (nonnegative) random variable X is said to have the ***memoryless property*** if, for any $s > 0$ and $t > 0$,

$$P(X > s + t \,|\, X > t) \;=\; P(X > s).$$

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- A (nonnegative) random variable X is said to have the ***memoryless property*** if, for any $s > 0$ and $t > 0$,

$$P(X > s + t \,|\, X > t) \;=\; P(X > s).$$

If X is a **waiting time** in minutes, say, this says that the probability that you'll need to wait **an additional s minutes**, given that you've **already waited t minutes**, doesn't depend on how long you've already waited (t).

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Memoryless Property:

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1. If $\boldsymbol{X} \sim$ **geometric**$(\boldsymbol{p})$, then X has the memoryless property.
2. If $\boldsymbol{X} \sim$ **exponential**$(\boldsymbol{\lambda})$, then X has thememoryless property.

Proof (for the exponential case): If

$$X \sim \text{exponential}(\lambda),$$

then (for $x \geq 0$)

$$F(x) = 1 - e^{-\lambda x}.$$

Thus

$$P(X > s + t \,|\, X > t) \quad = \quad \frac{P(\{X > s + t\} \cap \{X > t\})}{P(X > t)}$$

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\begin{aligned}
P(X > s+t \,|\, X > t) &= \frac{P(\{X > s+t\} \cap \{X > t\})}{P(X > t)} \\
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Example

Suppose again that you're trying to hail a taxi cab, and

$$X = \text{The amount of time (in minutes) that you have to wait.}$$

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X = The amount of time (in minutes) that you have to wait.

Suppose again that

$$X \sim \textbf{exponential}(0.1)$$

Then the (conditional) **probability** that you'll have to wait **an additional ten minutes**, given that you've **already waited fifteen minutes**, is

$$P(X > 10 + 15 \,|\, X > 15) \;\; = \;\; P(X > 10)$$

$$\begin{aligned} P(X > 10 + 15 \,|\, X > 15) &= P(X > 10) \\ &= \mathbf{0.3679} \end{aligned}$$

(from a previous example).