MTH 3210
Exam II Study Guide

The exam covers Slides 6-14 (but not linear combinations of random variables), Homeworks 4-7, and Sections 3.1-3.6, 4.1-4.4, 5.3-5.4, in the book. Exam problems will be similar to examples done in class and homework problems.

1. **Random variables introduction**
   - Distinguish discrete from continuous random variables
2. **Discrete random variables**
   - Know the two properties that **pmf**s satisfy: $p(x) \geq 0$, $\sum p(x) = 1$.
   - Use **pmf**s to find probabilities involving discrete random variables: $p(x) = P(X = x)$.
   - For a discrete random variable with a given **pmf**, know how to compute and interpret:
     - The expected value $E(X)$.
     - The expected value $E[h(X)]$ of a function $h(X)$ of the random variable
     - The expected value of $E(aX + b)$ a linear function $aX + b$ the random variable.
     - The variance $V(X)$ and standard deviation $SD(X)$
     - The variance and standard deviation $V(aX+b)$ and $SD(aX+b)$ of a linear function of the random variable
3. **Special discrete random variables**
   - Be able to **recognize** each of the following types of random variables, know what each one's **pmf** is, be able to use the **pmf** to compute probabilities, and know what each one's **mean**, **variance**, and **standard deviation** are.
     - Bernoulli with parameter p
     - Geometric with parameter p
     - Binomial with parameters n and p
     - Poisson with parameter μ
   - **Poisson process**: Know how to compute probabilities involving a random variable X in a Poisson process with rate parameter α.
4. **Continuous random variables**
   - Know the two properties that **pdf**s satisfy: $f(x) \geq 0$, $\int f(x)\,dx = 1$.
   - Use **pdf**s to find probabilities involving continuous random variables: $\int_a^b f(x)\,dx = P(a \leq X \leq b)$.
   - For a continuous random variable with a given **pdf**, know how to compute and interpret:
     - The expected value $E(X)$.
     - The expected value $E[h(X)]$ of a function $h(X)$ of the random variable
     - The expected value of $E(aX + b)$ a linear function $aX + b$ the random variable.
     - The variance $V(X)$ and standard deviation $SD(X)$
     - The variance and standard deviation $V(aX+b)$ and $SD(aX+b)$ of a linear function of the random variable

5. **Special continuous random variables**

   - Be able to **recognize** each of the following types of random variables, know what each one's **pdf** is, be able to use the **pdf** (or **table** in the case of a normal random variable) to compute probabilities, and know what each one's **mean**, **variance**, and **standard deviation** are.
     - Uniform distribution with parameters (endpoints) A, B.
     - Exponential distribution with parameter λ.
     - Normal distribution with parameters μ and σ:
       * Know that it's two parameters μ and σ are it's mean (expected value) and standard deviation.
       * Know the Empirical Rule (68-95-99.7 Rule) for the normal distribution.
       * Know that if X is normal and if $Y = aX + b$, then Y is also normal, and know what the mean and standard deviation of Y are.
       * Know how to find **probabilities** such as $P(X \leq b)$, $P(X > a)$, and $P(a \leq X \leq b)$ (by standardizing a and b and then using Table A.3).
       * Know how to find **percentiles** of any normal distribution (by finding and "un-standardizing" the corresponding percentile of the N(0,1)).
       * ***Z*-scores**: know how to compute them, interpret them, and use them as measures of relative standing.

6. **Cumulative distribution functions (cdfs)**

   - Know what they are (i.e. $F(x) = P(X \leq x)$).
   - Know how to find $F(x)$ from a given **pdf** f(x) (i.e. $F(x) = \int_{-\infty}^{x} f(y)\,dy$) .
   - Know how to find the **pdf** $f(x)$ given a **cdf** $F(x)$ (i.e. $f(x) = \frac{d}{dx}F(x)$).
   - Know how to find **probabilities** such as $P(X \leq b)$, $P(X > a)$, and $P(a \leq X \leq b)$ using the **cdf** $F(x)$ (e.g. $P(a \leq X \leq b) = F(b) - F(a)$).
   - Know how to find a **percentile** η_p of a continuous distribution using the **cdf** (e.g. the 75th percentile $\eta_{0.75}$ is found by solving $F(\eta) = 0.75$ for η).
   - Know the **cdf**s of the uniform and exponential distributions.

7. **Statistics and sampling distributions**

   - **Statistic**: know the definition.
   - Sampling distribution of a statistic
     - Know what a sampling distribution is.
     - Know the mean $\mu_{\bar{x}}$ and standard error $\sigma_{\bar{x}}$ of the sampling distribution of $\bar{X}$.
     - Know that when a sample $X_1, X_2, \ldots, X_n$ comes from a N(μ, σ) population, $\bar{X}$ follows a N$(\mu_{\bar{x}}, \sigma_{\bar{x}})$ distribution.
     - Know how to find probabilities associated with $\bar{X}$, e.g. $P(a \leq \bar{X} \leq b)$, when the sample $X_1, X_2, \ldots, X_n$ is from a N(μ, σ) population.
     - **Central Limit Theorem (CLT)**:
       * Know what the **CLT** says about the sampling distribution of $\bar{X}$ when n is large no matter what the distribution of the population is.
       * Know how to use the **CLT** to find probabilities such as $P(a \leq \bar{X} \leq b)$ approximately when n is large no matter what the distribution of the population is.