Probability and Statistics

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Topics

Objectives

Objectives:

- Carry out:
  - Two-sample z test for the difference between two normal means $\mu_1 - \mu_2$ when σ_1 and σ_2 are known.
  - Two-sample z test for the difference between two general population means $\mu_1 - \mu_2$ when m and n are large.
  - Two-sample t test for the difference between two normal means $\mu_1 - \mu_2$ when σ_1 and σ_2 are unknown.

Test for the Difference Between Two Normal Means $\mu_1 - \mu_2$ when σ_1 and σ_2 are Known (9.1)

- Suppose
  1. $X_1, X_2, \ldots, X_m$ are a random sample from a **N**(μ_1, σ_1) population.
  2. $Y_1, Y_2, \ldots, Y_n$ are a random sample from a **N**(μ_2, σ_2) population.
  3. The population meansμ_1 and μ_2 **are unknown** but the standard deviations σ_1 and σ_2 **are known.**
  4. The X and Y samples are **independent** of each other.

Test for the Difference Between Two Normal Means $\mu_1 - \mu_2$ when σ_1 and σ_2 are Known (9.1)

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  1. $X_1, X_2, \ldots, X_m$ are a random sample from a **N**$(\boldsymbol{\mu_1}, \boldsymbol{\sigma_1})$ population.
  2. $Y_1, Y_2, \ldots, Y_n$ are a random sample from a **N**$(\boldsymbol{\mu_2}, \boldsymbol{\sigma_2})$ population.
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  2. $Y_1, Y_2, \ldots, Y_n$ are a random sample from a **N**(μ_2, σ_2) population.
  3. The population meansμ_1 and μ_2 **are unknown** but the standard deviations σ_1 and σ_2 **are known**.
  4. The X and Y samples are **independent** of each other.
- We'll see how to test whether μ_1 and μ_2 are different from each other.

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- When σ_1 and σ_2 are known, the appropriate test is called the ***two-sample z test***
- The difference $\bar{X} - \bar{Y}$ between the two sample means is an **estimator** of the (unknown) difference between the population means $\mu_1 - \mu_2$.
- $\bar{X} - \bar{Y}$ is a difference between two **normal** random variables, so it too follows a **normal distribution.**

Normality of $\bar{X} - \bar{Y}$: If $X_1, X_2, \ldots, X_m$ is a random sample from a N$(\mu_1,\, \sigma_1)$ distribution, and $Y_1, Y_2, \ldots, Y_n$ is a random sample from a N$(\mu_2,\, \sigma_2)$ distribution, and the two samples are **independent** of each other, then

$$\bar{X} - \bar{Y} \;\sim\; \mathrm{N}\left(\mu_1 - \mu_2,\ \sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}\right).$$

Normality of $\bar{X} - \bar{Y}$: If $X_1, X_2, \ldots, X_m$ is a random sample from a N(μ_1, σ_1) distribution, and $Y_1, Y_2, \ldots, Y_n$ is a random sample from a N(μ_2, σ_2) distribution, and the two samples are **independent** of each other, then

$$\bar{X} - \bar{Y} \sim \text{N}\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}\right).$$

It follows that

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/m + \sigma_2^2/n}} \sim \text{N}(0, 1).$$

Proof: From Slides 14,

$$\bar{X} \sim \mathsf{N}\left(\mu_1, \frac{\sigma_1}{\sqrt{m}}\right) \qquad \text{and} \qquad \bar{Y} \sim \mathsf{N}\left(\mu_2, \frac{\sigma_2}{\sqrt{n}}\right).$$

Furthermore, $\bar{X} - \bar{Y}$ is a linear combination of $\bar{X}$ and $\bar{Y}$ (which are independent because the two samples are), so (also from Slides 14)

$$\bar{X} - \bar{Y} \sim \mathsf{N}\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}\right).$$

- We're seeking to "disprove" the claim that μ_1 is equal to μ_2, so the **null hypothesis** is that they *are* equal.

Null Hypothesis:

$$H_0 : \mu_1 - \mu_2 \ = \ 0$$

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Null Hypothesis:

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(H_0 could also be written as $H_0 : \mu_1 = \mu_2$.)

- The **alternative hypothesis** will depend on what we're trying to "prove":

Alternative Hypothesis: The alternative hypothesis will be one of

1. $H_a : \mu_1 - \mu_2 > 0$ (**one-sided, upper-tailed**)
2. $H_a : \mu_1 - \mu_2 < 0$ (**one-sided, lower-tailed**)
3. $H_a : \mu_1 - \mu_2 \neq 0$ (**two-sided, two-tailed**)

depending on what we're trying to verify using the data.

- The **test statistic** for the ***two-sample z test for $\mu_1 - \mu_2$*** is

Two-Sample Z Test Statistic:

$$Z = \frac{\bar{X} - \bar{Y} - 0}{\sqrt{\sigma_1^2/m + \sigma_2^2/n}}$$

When H_0 is true, $Z \sim \mathrm{N}(0, 1)$.

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- Z measures how many standard errors $\bar{X} - \bar{Y}$ is away from **zero**.

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  1. Z will be approximately **zero** if $\mu_1 - \mu_2 = 0$.
  2. It will be **positive** if $\mu_1 - \mu_2 > 0$.
  3. It will be **negative** if $\mu_1 - \mu_2 < 0$.

1. **Large positive** values of Z provide **evidence against H_0 in favor of** $H_a : \mu_1 - \mu_2 > 0$.
2. **Large negative** values of Z provide **evidence against H_0 in favor of** $H_a : \mu_1 - \mu_2 < 0$.
3. **Large positive *and* large negative** values of Z provide **evidence against H_0 in favor of** $H_a : \mu_1 - \mu_2 \neq 0$.

- Recall that

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/m + \sigma_2^2/n}} \sim \text{N}(0, 1).$$

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$$\frac{\bar{X} - \bar{Y} - (\boldsymbol{\mu_1} - \boldsymbol{\mu_2})}{\sqrt{\sigma_1^2/m + \sigma_2^2/n}} \sim \text{N}(0, 1).$$

- It follows that **if $\boldsymbol{H_0}$ is true** (so $\mu_1 - \mu_2 = 0$),

$$\frac{\bar{X} - \bar{Y} - \mathbf{0}}{\sqrt{\sigma_1^2/m + \sigma_2^2/n}} \sim \text{N}(0, 1).$$

Sampling Distribution of the Test Statistic Under H_0:
If Z is the two-sample Z test statistic, then when

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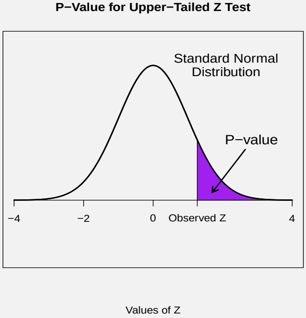
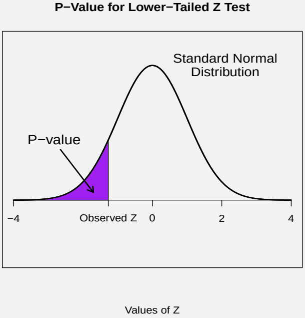
- The ***p-value*** is the probability that just by chance (under H_0) we'd get a test statistic value as far from zero, in the direction predicted by H_a, as the observed value.

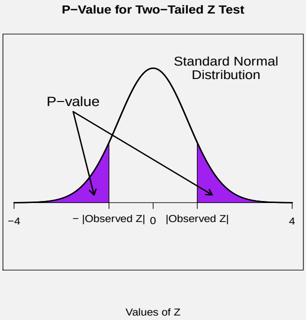
1. **P-value** = Area to the **right** of the observed z if the alternative hypothesis is $H_a : \mu_1 - \mu_2 > 0$.



1. **P-value** = Area to the **left** of the observed z if the alternative hypothesis is $H_a : \mu_1 - \mu_2 < 0$.



1. **P-value** = Area to the **left** of $-|z|$ **and right** of $|z|$ if the alternative hypothesis is $H_a : \mu_1 - \mu_2 \neq 0$.



Two-Sample Z Test for $\mu_1 - \mu_2$ when σ_1 and σ_2 are Known

Assumptions: The data $x_1, x_2, \ldots, x_m$ are a random sample from a N(μ_1, σ_1) distribution and $y_1, y_2, \ldots, y_n$ are a random sample from a N(μ_2, σ_2) distribution, where σ_1 and σ_2 are known. Also, the two samples are independent.

Null hypothesis: $H_0 : \mu_1 - \mu_2 = 0$.

Test statistic value: $z = \frac{\bar{x} - \bar{y} - 0}{\sqrt{\sigma_1^2/m + \sigma_2^2/n}}$.

Decision rule: Reject H_0 if p-value $< \alpha$.

Alternative hypothesis	**P-value** = area under N$(0, 1)$ distribution:
$H_a : \mu_1 - \mu_2 > 0$	to the right of z
$H_a : \mu_1 - \mu_2 < 0$	to the left of z
$H_a : \mu_1 - \mu_2 \neq 0$	to the left of $-\lvert z \rvert$ and right of $\lvert z \rvert$

Large Sample Test for the Difference Between Two General Population Means $\mu_1 - \mu_2$ (9.1)

- When the **sample sizes** m **and** n **are both large**, two things are true:

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  1. Regardless of the shape of the populations, by the Central Limit Theorem,

$$Z = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/m + \sigma_2^2/n}} \sim \text{N}(0, 1) \qquad \text{(approximately).}$$

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2. The **sample standard deviations** S_1 and S_2 will remain fairly **constant** from one sample to the next, and **approximately equal to** σ_1 **and** σ_2.

- As a consequence, **when n is large**,

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- It follows that **if H_0 is true** (so $\mu_1 - \mu_2 = 0$),

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- Thus we can use this as our **test statistic** in a ***two-sample z test for $\mu_1 - \mu_2$***.

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When H_0 is true, $Z \sim$ N$(0, 1)$.

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- The ***p-value*** is the appropriate tail area under the $\mathrm{N}(0,\ 1)$ curve.
- In practice, m and n are **large enough** if $m > 40$ and $n > 40$.

Two-Sample Z Test for $\mu_1 - \mu_2$ when m and n and n are Large

Assumptions: The data $x_1, x_2, \ldots, x_m$ are a random sample from *any* distribution whose mean and standard deviation are μ_1 and σ_1 and $y_1, y_2, \ldots,$ y_n are a random sample from *any* distribution whose mean and standard deviation are μ_2 and σ_2. Also, the two samples are independent of each other.

Null hypothesis: $H_0 : \mu_1 - \mu_2 = 0$.

Test statistic value: $z = \frac{\bar{x} - \bar{y} - 0}{\sqrt{s_1^2/m + s_2^2/n}}$.

Decision rule: Reject H_0 if p-value $< \alpha$.

Alternative hypothesis	**P-value** = area under N$(0, 1)$ distribution:
$H_a : \mu_1 - \mu_2 > 0$	to the right of z
$H_a : \mu_1 - \mu_2 < 0$	to the left of z

Test for the Difference Between Two Normal Means $\mu_1 - \mu_2$ When σ_1 and σ_2 are Unknown (9.2)

- It can be shown that if $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples from two **normal** populations, the random variable

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{S_1^2/m + S_2^2/n}} \sim t(\nu),$$

a t ***distribution*** with ν ***degrees of freedom***, where

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$$\nu = \frac{\left(\frac{s_1^2}{m} + \frac{s_2^2}{n}\right)^2}{\frac{(s_1^2/m)^2}{m-1} + \frac{(s_2^2/n)^2}{n-1}},$$

which should be truncated **down** to the nearest integer.

- It follows that **if H_0 is true** (so $\mu_1 - \mu_2 = 0$),

$$\frac{\bar{X} - \bar{Y} \ - \ 0}{\sqrt{S_1^2/m + S_2^2/n}} \ \sim \ t(\nu),$$

- Thus we can use this as our **test statistic** in a ***two-sample t test for $\mu_1 - \mu_2$.***

- The **test statistic** for the ***two-sample*** t ***test for*** $\mu_1 - \mu_2$ is

Two-Sample t Test Statistic:

$$T = \frac{\bar{X} - \bar{Y} - 0}{\sqrt{S_1^2/m + S_2^2/n}}.$$

When H_0 is true, $T \sim t(\nu)$.

Sampling Distribution of the Test Statistic Under H_0:
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- The ***p-value*** is the appropriate tail area under the t(ν) curve.

Two-Sample t Test for $\mu_1 - \mu_2$

Assumptions: The data $x_1, x_2, \ldots, x_m$ are a random sample from a N(μ_1, σ_1) distribution and $y_1, y_2, \ldots, y_n$ are a random sample from a N(μ_2, σ_2) distribution. Also, the two samples are independent of each other.

Null hypothesis: $H_0 : \mu_1 - \mu_2 = 0$.

Test statistic value: $t = \frac{\bar{x} - \bar{y} - 0}{\sqrt{s_1^2/m + s_2^2/n}}$.

Decision rule: Reject H_0 if p-value $< \alpha$.

Alternative hypothesis	**P-value** = area under $t(\nu)$ distribution*:
$H_a : \mu_1 - \mu_2 > 0$	to the right of t
$H_a : \mu_1 - \mu_2 < 0$	to the left of t
$H_a : \mu_1 - \mu_2 \neq 0$	to the left of $-\|t\|$ and right of $\|t\|$

* $t(\nu)$ is the t distribution with d.f. ν given a few slides back.

Example

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Twelve workers are randomly assigned to two groups of **six** workers **each.**

The first group measures the strength of the fabric using **Work Sequence 1** and the second measures it using **Work Sequence 2**.

The following data are the **completion times** (in **seconds**) for each group:

Work Sequence 1	Work Sequence 2
220	247
235	223
214	215
197	219
206	207
214	236

The **summary statistics** for the two groups are:

Work Sequence 1	Work Sequence 2
$m = 6$	$n = 6$
$\bar{x} = 214.3$	$\bar{y} = 224.5$
$s_1 = 12.9$	$s_2 = 14.6$

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$s_1 = 12.9$	$s_2 = 14.6$

We'll carry out a **two-sample t test** to decide **which work sequence, if any, is faster**.

The **hypotheses** are

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

where μ_1 and μ_2 are the true (unknown) population mean completion times.

The observed **test statistic** is

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Thus the observed difference between **sample mean** completion times, $\bar{x} - \bar{y} = -\mathbf{10.2}$, is about **1.28 standard errors below zero.**

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Thus the observed difference between **sample mean** completion times, $\bar{x} - \bar{y} = -10.2$, is about **1.28 standard errors below zero.**

The **p-value** is the **probability** that we'd get a t value this far away from zero (in either direction) by chance **if** there was **no difference** in the **population means** μ_1 and μ_2.

Under H_0, the test statistic would follow a $t(\nu)$ **distribution** with **degrees of freedom**

$$\nu = \frac{\left(\frac{s_1^2}{m} + \frac{s_2^2}{n}\right)^2}{\frac{(s_1^2/m)^2}{m-1} + \frac{(s_2^2/n)^2}{n-1}} = \frac{\left(\frac{12.9^2}{6} + \frac{14.6^2}{6}\right)^2}{\frac{(12.9^2/6)^2}{6-1} + \frac{(14.6^2/6)^2}{6-1}} = \mathbf{9.8},$$

which we round **down** to **9**.

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From the **two tail** areas of the $t(9)$ **distribution**, to the **left** of **-1.28** and **right** of **1.28**,

$$\textbf{p-value} = 2(0.116) = \mathbf{0.232}.$$

Thus we'd get a result like the one we got **23.2%** of the time **even if** the **population mean** completion times μ_1 and μ_2 were equal.

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Using a **level of significance** $\alpha = 0.05$, the **decision rule** is

Reject H_0 if p-value < 0.05.
Fail to reject H_0 if p-value ≥ 0.05.

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Using a **level of significance** $\alpha = 0.05$, the **decision rule** is

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Fail to reject H_0 if p-value ≥ 0.05.

Because $0.232 \geq 0.05$, we **fail to reject** H_0.

Thus we'd get a result like the one we got **23.2%** of the time **even if** the **population mean** completion times μ_1 and μ_2 were equal.

Using a **level of significance** $\alpha = 0.05$, the **decision rule** is

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The observed difference can be explained by chance variation (sampling error).