Introduction to Statistics

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Topics

1 Finding the x Value for a Specified Proportion

Objectives

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- Interpret percentiles of a normal distribution.
- Obtain percentiles of a normal distribution.

Finding the x Value for a Specified Proportion (6.2, 6.3)

Percentiles of the Normal Distribution (6.3)

- Sometimes we'll need to find the value of a variable x **below which** a specified **proportion** (or **percentage**) of the population lies.

Finding the x Value for a Specified Proportion (6.2, 6.3)

Percentiles of the Normal Distribution (6.3)

- Sometimes we'll need to find the value of a variable x **below which** a specified **proportion** (or **percentage**) of the population lies.
- An x value to the left of which a specified **proportion** of a normal distribution lies is called a ***percentile*** of the distribution.

Example

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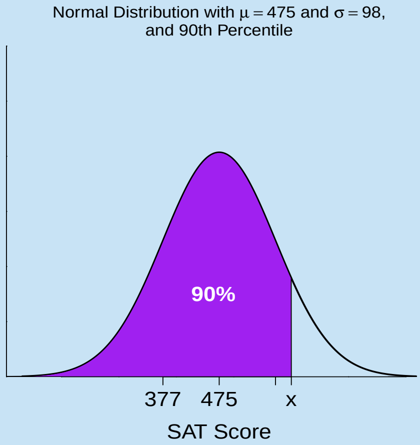
It's the value marked x on the horizontal axis on the next slide.


Normal Distribution with μ = 475 and σ = 98,
and 90th Percentile
90%
377
475
x
SAT Score

Finding a Percentile: To find the x value (*percentile*) that has a specified proportion to its *left* under a normal curve with mean μ and standard deviation σ:

1. **Search Table II** for the specified proportion, then **determine the z-score** associated with that proportion.
2. **"Unstandardize"** the z-score using:

$$\boldsymbol{x = \mu + z\sigma}$$

(This formula was obtained by solving $z = \frac{x-\mu}{\sigma}$ for x.)

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The closest we can get is **0.8997**, and the associated z value (from the table margin) is

$$z = \mathbf{1.28}.$$

Next, we "unstandardize" this z value, which converts it from standard units to an SAT score:

$$\begin{aligned} x &= \mu + z\sigma \\ &= 475 + 1.28(98) \\ &= \mathbf{600}. \end{aligned}$$

Thus the **90th percentile** of the distribution of SAT scores is

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This is the value marked x on the horizontal axis under the normal curve a few slides back.

Exercise

Heights of men aged 20 to 29 follow a **normal** distribution with **mean 69.3** inches and **standard deviation 2.8** inches.

a) Find the height x that separates the **shortest 2.5%** of men from the rest (i.e. the **2.5th percentile** of the distribution).

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b) Find the height x that separates the **tallest 2.5%** from the rest (i.e. the **97.5th percentile** of the distribution).

c) **Between** what **two heights** do the **middle 95%** of men's heights lie?

Exercise

A machine that cuts corks for wine bottles produces corks whose diameters vary a bit. The distribution of cork diameters is a **normal** distribution with **mean 3.0** cm and **standard deviation 0.1** cm.

a) Find the diameter x below which only **5%** of corks' diameters lie (i.e. the **5th percentile** of the distribution).

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a) Find the diameter x below which only **5%** of corks' diameters lie (i.e. the **5th percentile** of the distribution).

b) Find the diameter x above which only **5%** of all corks' diameters lie (i.e. the **95th percentile** of the distribution).

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a) Find the diameter x below which only **5%** of corks' diameters lie (i.e. the **5th percentile** of the distribution).

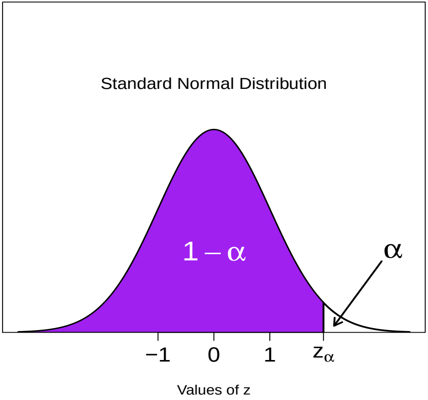
b) Find the diameter x above which only **5%** of all corks' diameters lie (i.e. the **95th percentile** of the distribution).

c) **Between** what **two diameters** do the **middle 90%** of corks' diameters lie?

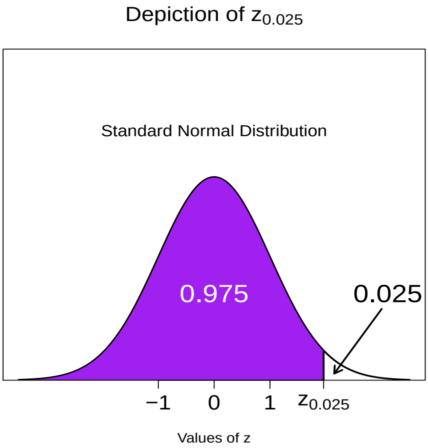
The z_α Notation

- We use z_α to denote the z**-score** that has a **proportion** α to its ***right*** under the ***standard normal curve***, as depicted on the next slide.

Depiction of z_α



- For example, $z_{0.025}$ has a proportion **0.025** to its **right** under the standard normal curve, as shown on the next slide.


Depiction of $z_{0.025}$
Standard Normal Distribution
0.975
0.025
−1
0
1
$z_{0.025}$
Values of z

- Note that the proportion (area) to the ***left*** of $z_{\mathbf{0.025}}$ is

$$1 - 0.025 \; = \; \mathbf{0.975},$$

so $z_{\mathbf{0.025}}$ is the **97.5th percentile** of the **standard normal distribution.**

Finding a z_α Value: To find a z_α value, which has a proportion α to its **right** under the *standard normal* curve, **search Table II** for the proportion $1 - \alpha$, then **determine the z-score** associated with that proportion.

Exercise

Use **Table II** to determine the values of $z_{0.005}$, $z_{0.025}$, and $z_{0.05}$.