

# Introduction to Statistics

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September 2, 2019

## Topics

1 The Empirical Rule

2 Z-Scores

## Objectives

### Objectives:

- Apply the Empirical Rule to data that follow a bell-shaped distribution.
- Compute and interpret  $z$ -scores.
- Use  $z$ -scores as measures of the relative standing of individuals from two populations.

## The Empirical Rule (3.3)

- The **Empirical Rule** (or **68-95-99.7 Rule**) says that for data that follow a **bell-shaped** distribution:
  1. **68%** of the observations fall within **one standard deviation** of the **mean**.
  2. **95%** of the observations fall within **two standard deviations** of the **mean**.
  3. **99.7%** of the observations fall within **three standard deviations** of the **mean**.

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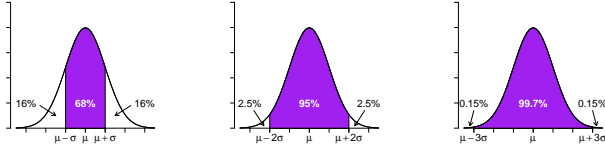
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Exercise

Data on the heights of women aged 20 to 29 show that the **mean** height is **64** inches and the **standard deviation** of the heights is **2.7** inches. A histogram shows that the heights have a **bell-shaped** distribution

- a) What percentage of women are **between 61.3** and **66.7** inches tall?
- b) What percentage of women are **between 58.6** and **69.4** inches tall?

- c) What percentage of women are **shorter** than **66.7** inches?
- d) What percentage of women are **taller** than **72.1** inches?

Z-Scores (3.5)

Calculating a Z-Score

- If we know the population mean  $\mu$  and standard deviation  $\sigma$  of a variable, then for any value  $x$  of that variable, the **z-score** for that value (also called its **standardize value**) is:

Z-Score:

$$z = \frac{x - \mu}{\sigma}$$

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- If we don't know  $\mu$  and  $\sigma$ , but we have a sample from the population, the **z-score (standardize value)** is:

**Z-Score (when  $\mu$  and  $\sigma$  are unknown):**

$$z = \frac{x - \bar{x}}{s}$$

### Properties and Interpretation of Z-Scores

1. The *z-score* indicates how many **standard deviations above or below the mean** an individual  $x$  value lies. These units are called **standard units**.
2. If an individual's  $x$  value is above the mean, their **z-score** will be **positive**. If it's below the mean, their **z-score** will be **negative**. If it's equal to the mean, their **z-score** will be **zero**.

3. The **farther away from zero** a *z-score* is, the more **"atypical"** the individual is. In particular, for data with a bell-shaped distribution, the Empirical Rule says:
  - A *z-score* larger than 2.0 or smaller than -2.0 occurs only 5% of the time (i.e. **not very often**).
  - A *z-score* larger than 3.0 or smaller than -3.0 occurs only 0.3% of the time (i.e. **very rarely**).

#### Exercise

The study of women's heights also looked at heights of men between the ages 20 to 29 years. It found that the **mean** height of men is **69.3** inches with a **standard deviation** of **2.8** inches.

- a) Suppose a man is **72** inches (six feet) tall. **Calculate** and **interpret** his **z-score**.
- b) Suppose a man is **84** inches (seven feet) tall. **Calculate** and **interpret** his **z-score**.

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### The Z-Score as a Measure of Relative Standing

- A  $z$ -score indicates where an individual  $x$  value stands **relative to the rest of the population**. So we can compare individuals from two **different** populations by comparing their  $z$ -scores.

#### Exercise

Recall that the **mean** height for **women** is **64** inches with a **standard deviation** of **2.7** inches, and the **mean** height for **men** is **69.3** inches with a **standard deviation** of **2.8** inches.

If a **woman** is **69** inches tall, and a **man** is **72** inches tall, who's taller *relative to his or her gender*?

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