Introduction to Statistics

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September 2, 2019

Notes

Topics

Notes

Objectives

Objectives:

- Identify the sample space of a chance experiment.
- Interpret an event as a group of outcomes in the sample space.
- Interpret probabilities as long-run proportions, and know the properties that all probabilities satisfy.
- Compute probabilities of events when outcomes are equally likely.

Notes

(objectives cont'd):

- Form new events using ”not”, ”and”, and ”or”.
- Recognize mutually exclusive events.
- Produce a Venn diagram.
- Apply the Complementation Rule and the Special Addition Rule to obtain probabilities.
- Apply the General Addition Rule to obtain a probability.

Notes

Probability Basics, Events (4.1, 4.2)

Chance Experiments, Sample Spaces, and Events

- A ***chance experiment*** is any action or process whose **outcome is uncertain**.

Example

Some examples of **chance experiments**:

- Tossing a coin
- Rolling a six-sided die
- Randomly selecting a student from a university
- Drawing a card from a shuffled deck

Notes

- The ***sample space*** of a chance experiment is the **collection of all the possible outcomes**.

Example

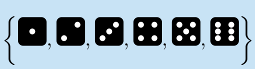
Here are the **sample spaces** for some of the chance experiments of the last example.

- If we toss a coin, the **sample space** consists of the two outcomes, heads and tails, which we'll write as

$$\textbf{Sample Space} = \{H, T\}.$$

Notes

- If we roll a six-sided die, the **sample space** is comprised of the six outcomes, which we write as

Sample Space =  .

Notes

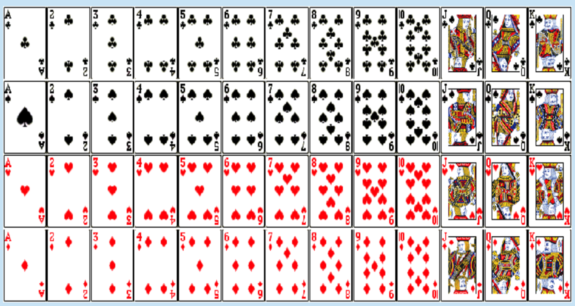
- If we randomly select a student from a university, the **sample space** is the entire list of students at the university (i.e. the set of "names in the hat"):

Sample Space = {Stephanie Lawson,
Jeffrey Miller,
Angela DuPont,
⋮
Melissa Jordan
Karl Stephenson}

Notes

- If we draw a card from a deck, the **sample space** is the entire set of 52 cards in the deck, as shown on the next slide.

Notes



Notes

- An ***event*** is **a single outcome or a group of outcomes** from within the sample space.

  We'll use **capital letters** (e.g. A, B, C, E, F, etc.) to denote **events**.

Example

Here are some **events** associated with the chance experiments of the previous two examples.

- Toss a coin. Consider the **event A** defined as

  A = The event that the coin lands on heads.

  Then **A** consists of the single outcome, which we write as

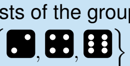
  A = {  }.

Notes

- Roll a six-sided die. Consider the **event B** defined as

  B = The event that the die lands on an even number.

  Then **B** consists of the group of the three outcomes

  B = {  }

Notes

- Randomly choose a student from a university, and define the **event E** to be

  E = The event that the student is female.

  Then **E** consists of the group of all the female students at the university, i.e.

  $$E = \{\text{Stephanie Lawson}, \text{Angela DuPont}, \ldots, \text{Melissa Jordan}\}$$

Notes

Definition of Probability

- The ***probability*** of an event A, denoted **P(A)**, is defined to be the **long-run proportion** of times A occurs.

  So, upon repeating the chance experiment a **very large** number of times,

  $$P(A) = \frac{\text{Number of times A occurs}}{\text{Number of times the chance experiment is repeated}}$$

Notes

Example

Toss a coin repeatedly. In the **long-run**, the coin will land (H) (heads) **50%** the time. So

$$P\left(\text{(H)}\right) = \mathbf{0.5}$$

Notes

Probability for Equally Likely Outcomes

- When the outcomes in the sample space are **equally likely**, **P(A)** is just the **fraction** of outcomes that comprise A.

Probability for Equally Likely Outcomes (f/N Rule)
Suppose an experiment has N possible outcomes, all equally likely. An event A that's comprised of f of the outcomes has probability

$$P(A) = \frac{\text{Number of outcomes that make up the event A}}{\text{Total number of outcomes in the sample space}} = \frac{f}{N}.$$

Notes

Example

If we toss a coin, the sample space has **two equally likely** outcomes, H and T, only **one** of which is heads. Thus, by the f/N Rule, the **probability** of **heads** is

$$P\left(H\right) = \frac{1}{2},$$

which is also the long-run proportion of times the coin will land heads.

Notes

Example

A jar contains **4 red** jelly beans, and **6 yellow** ones. You'll close your eyes and grab a jelly bean. Let

A = The event that you get a **red** jelly bean.

Then by the f/N Rule,

$$P(A) = \frac{4}{10} = \mathbf{0.4}.$$

Notes

Example

In the Fall 2016 semester, there were 19,800 students at MSU Denver, of whom 7,812 were students **of color**.

Suppose a student is randomly selected (so that each student is **equally likely** to be selected). Let

A = The event that the selected student is **of color**

Then by the f/N Rule,

$$P(A) = \frac{7,812}{19,800} = \mathbf{0.395}.$$

Notes

Basic Properties of Probabilities

- Probabilities satisfy the following properties.

Properties that All Probabilities Satisfy:

1. For any event A, $0 \leq P(A) \leq 1$.
2. If an event A cannot occur, then $P(A) = 0$.
3. If an event A is certain to occur, then $P(A) = 1$.

Notes

Relationships Among Events

- Suppose A and B are two events. Here are **new events** involving A and B:
  - The event ***(not A)*** is the event that "A doesn't occur." (Not A) is also called the ***complement*** of A.

    (Not A) consists of all the outcomes that are **not** in A.
  - The event ***(A & B)*** is the event that "A occurs and B occurs."

    (A & B) consists of the outcomes that are common to both A **and** B.
  - The event ***(A or B)*** is the event that "A occurs or B occurs (or both occur)."

    (A or B) consists of the outcomes that are either in A **or** in B or both.

Notes

Example

Roll a die. Let

A = The event that the die lands on an **even** number = {⚁, ⚃, ⚅}

B = The event that the die lands on a number **4 or higher** = {⚃, ⚄, ⚅}

Then

(Not A) = The event that the die lands on an **odd** number = {⚀, ⚂, ⚄}

Notes

A = The event that the die lands on an **even** number = {⚁, ⚃, ⚅}

B = The event that the die lands on a number **4 or higher** = {⚃, ⚄, ⚅}

Also,

(A & B) = The event that the die lands on an **even** number **and** it's **4 or higher** = {⚃, ⚅}

Notes

A = The event that the die lands on an **even** number = {⚁, ⚃, ⚅}

B = The event that the die lands on a number **4 or higher** = {⚃, ⚄, ⚅}

Finally,

(A *or* B) = The event that die lands on an **even** number **or** a number **4 or higher** = {⚁, ⚃, ⚄, ⚅}

Notes

Mutually Exclusive Events

- Two events A and B are ***mutually exclusive*** if **they can't *both* occur** (i.e. if they *share no outcomes in common*).

Notes

Example

Randomly select a student from a university. Let

A = The event that the selected student is **male**
B = The event that the selected student is **pregnant**

Then A and B are **mutually exclusive** because the selected student *cannot* be both male and pregnant, i.e. there are no students who belong to both the male group *and* the pregnant group.

Notes

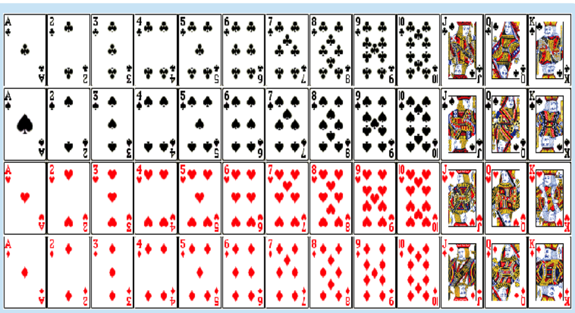
Example

Recall that in a standard deck of **52 cards**,

Each card is one of **13 ranks** (Ace, 2, 3, ..., Queen, King),

Each card also belongs to one of **four suits** (Clubs ♣, Spades ♠, Hearts ♡, and Diamonds ♢).

Notes



Notes

A card is to be randomly selected from a deck. Let

A = The event that the selected card is an **Ace**
B = The event that the selected card is a **Spade**

Then A and B are ***not* mutually exclusive** because the selected card *can* be both an Ace *and* a Spade, i.e. there's a card that belongs to both the Ace group *and* the Spade group.

Notes

Venn Diagrams

- A ***Venn diagram*** is a graphical depiction of the **sample space** as a **rectangle**, with **events** depicted as **circles** within the rectangle.

Notes

Exercise

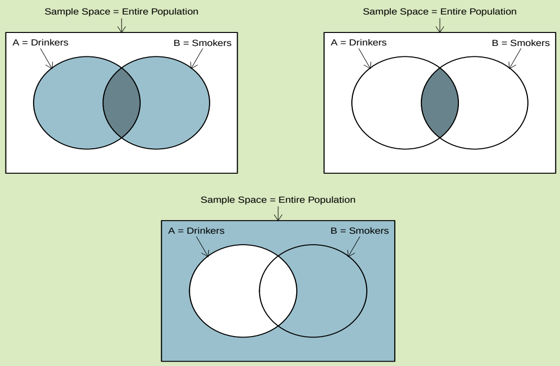
In a certain population, some of the people **drink** alcohol and some **smoke** cigarettes.

Consider randomly selecting an individual from this population. Let

A = The event that the selected person **drinks**
B = The event that the selected person **smokes**

a) **Describe** the event **(not A)** in words. Which **Venn diagram** on the next slide depicts this event as its shaded region?

Notes



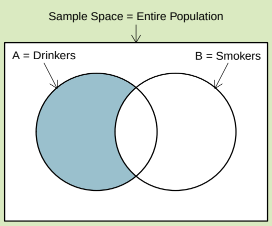
Notes

b) **Describe** the event **(A & B)** in words. Which **Venn diagram** on the previous slide depicts this event as its shaded region?

c) **Describe** the event **(A or B)** in words. Which **Venn diagram** on the previous slide depicts this event as its shaded region?

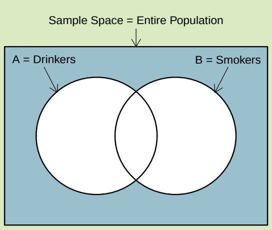
Notes

d) **Describe** in words the event that's shaded in the **Venn diagram** below.



Notes

e) **Describe** in words the event that's shaded in the **Venn diagram** below.



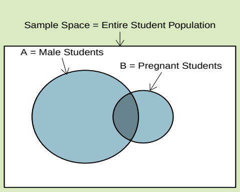
Notes

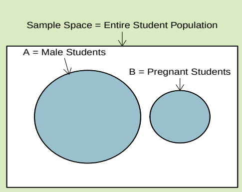
Exercise

Consider randomly select a student from a university. Let

A = The event that the selected student is **male**
B = The event that the selected student is **pregnant**

a) Which of the two **Venn diagrams** below most accurately depicts these **mutually exclusive** events?





Notes

b) In general, if two events A and B are **mutually exclusive**, how should they be depicted in a **Venn diagram**?

Notes

Some Probability Rules (4.3)

Complementation Rule and Special Addition Rule

- The following **probability rules** will help us calculate probabilities.

  1. The ***Complementation Rule*** says:

Complementation Rule: If A is **any** event, then

$$P(\textit{not } A) \;=\; 1 - P(A)$$

and also

$$P(A) \;=\; 1 - P(\textit{not } A)$$

Notes

- (cont'd)

  2. The ***Special Addition Rule*** says:

Special Addition Rule: If A and B are **mutually exclusive** events, then

$$P(A \textit{ or } B) \;=\; P(A) + P(B)$$

Notes

Exercise

Suppose a certain university has **1,000** students, of whom **475** are males and **100** are pregnant.

Consider randomly selecting an individual from this university, and let

A = The event that the selected person is **male**
B = The event that the selected person is **pregnant**

a) Use the **Complementation Rule** to find the probability that the selected student is ***not*** **male**.

b) Use the **Special Addition Rule** to find the probability that the selected student is **male *or* pregnant**. (The rule is applicable because A and B are **mutually exclusive**.)

Notes

Exercise

Suppose that in a population of **1,000** people, **300 drink** alcohol and **100 smoke** cigarettes.

Consider randomly selecting an individual from this population, and let

A = The event that the selected person drinks
B = The event that the selected person smokes

Is the **Special Addition Rule** applicable for finding the **probability** that the selected person **drinks *or* smokes**? **Why or why not**?

Notes

The General Addition Rule

- **When two events A and B *aren't* mutually exclusive, we *can't* use the Special Addition Rule** to find P(A *or* B).

  The next example shows what goes wrong.

Notes

Example

A library has **100 books**. **Seventy** of the books cover **statistics** and **40** cover **probability**.

We want to find the probability that a randomly selected book covers **statistics *or* probability**. Let

A = The event that the selected book covers **statistics**
B = The event that the selected book covers **probability**

Then $P(A) = \frac{70}{100}$ and $P(B) = \frac{40}{100}$.

Is it true that

$$P(A \text{ or } B) = P(A) + P(B) \quad \text{??????}$$

Notes

No!

P(A) and P(B) add up to $110/100$, but probabilities **can't** be bigger than 1.

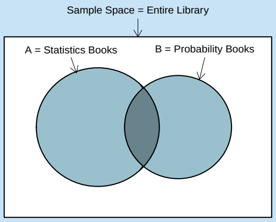
What went wrong?

Well, some of the books cover *both* topics. In other words, **A and B *aren't* mutually exclusive**.

So when we added $70/100$ to $40/100$, we **double-counted** the books that cover *both* topics.

The double-counted books are the ones in the overlap of the circles in the Venn diagram (next slide).

Notes



Notes

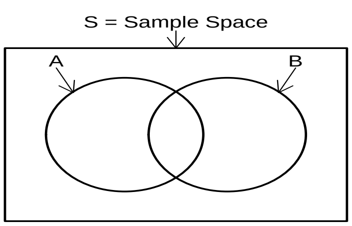
- The ***General Addition Rule*** applies even when events A and B **aren't** mutually exclusive.

General Addition Rule: For **any** two events A and B (**not necessarily mutually exclusive**),

$$P(A \text{ or } B) \;=\; P(A) + P(B) - P(A \,\&\, B)$$

Subtracting P(A & B) corrects for the fact that when A and B *aren't* mutually exclusive, some outcomes are **double-counted** when P(B) and P(A) are added together.

Notes



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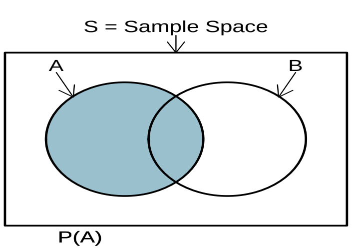

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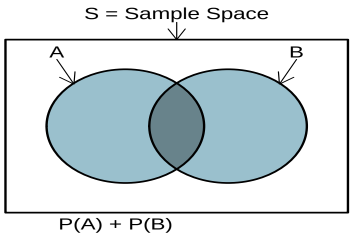

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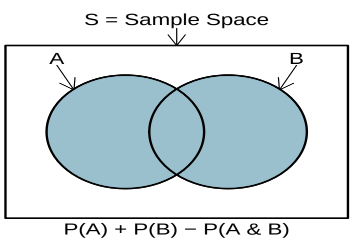


Notes

Exercise

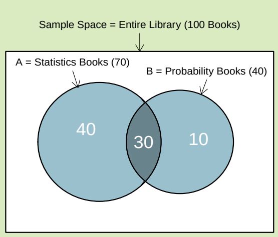
In the library that has **100** books, **70** of which cover *statistics* and **40** of which cover *probability*, suppose that **30** books cover ***both*** topics.

For a randomly selected book, let

A = The event that the selected book covers **statistics**
B = The event that the selected book covers **probability**

Refer to the following Venn diagram.

Notes



Notes

Notes

a) Use the **General Addition Rule** to find the probability that a randomly selected book covers **statistics *or* probability**.

b) Verify the answer to Part *a* via the Venn diagram.

Notes

Exercise

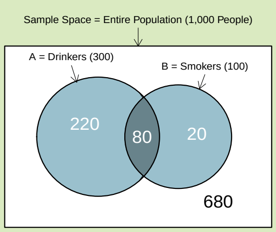
In a population of **1,000 people**, **300** drink alcohol and **100** smoke cigarettes. Suppose also that **80** people have **both** of these bad habits.

For a randomly selected individual, let

A = The event that the selected person **drinks**
B = The event that the selected person **smokes**

Refer to the following Venn diagram.

Notes



Notes

a) Use the **General Addition Rule** to find the probability that the selected person **drinks *or* smokes**.

b) Verify the answer to Part *a* via the Venn diagram.

Notes