Introduction to Statistics

Nels Grevstad

Metropolitan State University of Denver

ngrevsta@msudenver.edu

September 2, 2019

Topics

Objectives

Objectives:

- Produce and interpret a contingency table.
- Recognize independent and dependent events.
- Use the Multiplication Rule for Independent Events to compute probabilities.

Contingency Tables (4.4)

Contingency Tables

- Data from studies in which **two qualitative variables** are measured on each individual can be organized in a ***contingency table***.

Contingency Tables (4.4)

Contingency Tables

- Data from studies in which **two qualitative variables** are measured on each individual can be organized in a ***contingency table***.

  Each individual is **cross-classified** by the two qualitative variables, and the **frequencies** are reported in the table.

Contingency Tables (4.4)

Contingency Tables

- Data from studies in which **two qualitative variables** are measured on each individual can be organized in a ***contingency table***.

  Each individual is **cross-classified** by the two qualitative variables, and the **frequencies** are reported in the table.

  The next example illustrates.

Example

Each of **38,847** students at a large university was cross-classified according to **gender** and **student level**.

Example

Each of **38,847** students at a large university was cross-classified according to **gender** and **student level**.

The following **contingency table** summarizes the data:

		Student Level			
		Undergraduate	Professional	Graduate	
Gender	Male	18,208	249	4,436	
	Female	12,643	651	2,660	

Example

Each of **38,847** students at a large university was cross-classified according to **gender** and **student level**.

The following **contingency table** summarizes the data:

		Student Level			
		Undergraduate	Professional	Graduate	
Gender	Male	18,208	249	4,436	
	Female	12,643	651	2,660	

Each number in the table is a **frequency** (count).

Example

Each of **38,847** students at a large university was cross-classified according to **gender** and **student level**.

The following **contingency table** summarizes the data:

		Student Level			
		Undergraduate	Professional	Graduate	
Gender	Male	18,208	249	4,436	
	Female	12,643	651	2,660	

Each number in the table is a **frequency** (count).

For example, there are **18,208** male undergraduate students.

- The frequencies in body of the table occupy the so-called ***cells*** of the table.

- The frequencies in body of the table occupy the so-called ***cells*** of the table.
- Summing all of the cell frequencies gives the ***grand total frequency***.

- The frequencies in body of the table occupy the so-called ***cells*** of the table.
- Summing all of the cell frequencies gives the ***grand total frequency***.
- It's useful to also compute the so-called ***marginal total frequencies***, i.e. the total for each row and for each column.

Exercise

Here's the **contingency table** from the last example, but now showing the **grand total** and **marginal totals**:

		Student Level			
		Undergraduate	Professional	Graduate	Total
Gender	Male	18,208	249	4,436	22,893
	Female	12,643	651	2,660	15,954
	Total	30,851	900	7,096	38,847

a) How many **cells** are in the table?

Exercise

Here's the **contingency table** from the last example, but now showing the **grand total** and **marginal totals**:

		Student Level			
		Undergraduate	Professional	Graduate	Total
Gender	Male	18,208	249	4,436	22,893
	Female	12,643	651	2,660	15,954
	Total	30,851	900	7,096	38,847

a) How many **cells** are in the table?

b) How many **female graduate** students are at the university?

Exercise

Here's the **contingency table** from the last example, but now showing the **grand total** and **marginal totals**:

		Student Level			
		Undergraduate	Professional	Graduate	Total
Gender	Male	18,208	249	4,436	22,893
	Female	12,643	651	2,660	15,954
	Total	30,851	900	7,096	38,847

a) How many **cells** are in the table?

b) How many **female graduate** students are at the university?

c) How many **total graduate** students are there?

d) How many **total female** students are there?

d) How many **total female** students are there?

e) How many students are there in **total**?

- The next exercise illustrates how to turn a **two-variable qualitative data set** into a **contingency table**.

Exercise

Here's a (hypothetical) **two-variable data set** showing $n = 12$ peoples' **age group** (Young, Old) and **political affiliation** (Democrat, Republican):

Person	Age Group	Political Affiliation
1	Young	Democrat
2	Young	Democrat
3	Old	Republican
4	Young	Democrat
5	Old	Democrat
6	Young	Republican
7	Young	Democrat
8	Old	Democrat
9	Old	Republican
10	Old	Democrat
11	Young	Democrat
12	Young	Republican

Use the data from the previous slide to fill in the **contingency table** below:

		Political Affiliation		
		Democrat	Republican	Total
Age Group	Young			
	Old			
	Total			

Independent and Dependent Events (4.6)

Independent and Dependent Events

- Two events A and B are said to be ***independent*** if the occurrence of one has **no impact** on whether or not the other one occurs.

Independent and Dependent Events (4.6)

Independent and Dependent Events

- Two events A and B are said to be ***independent*** if the occurrence of one has **no impact** on whether or not the other one occurs.

  They're ***dependent*** if the occurrence of one affects whether or not the other one occurs.

Exercise

Decide if each of the following pairs of events are **independent** or **dependent.**

Exercise

Decide if each of the following pairs of events are **independent** or **dependent.**

a) For two tosses of a coin,

A = The event that the coin lands H on **first toss**

B = The event that the coin lands H on **second toss**

Exercise

Decide if each of the following pairs of events are **independent** or **dependent**.

a) For two tosses of a coin,

A = The event that the coin lands H on **first toss**
B = The event that the coin lands H on **second toss**

b) For a couple planning on having two children,

A = The event that the **first child** is a **girl**
B = The event that the **second child** is a **girl**

c) A car pulls into a metered parking spot.

A = The event that the driver **doesn't put money in the meter**

B = The event that the driver **gets a parking ticket**

c) A car pulls into a metered parking spot.

A = The event that the driver **doesn't put money in the meter**

B = The event that the driver **gets a parking ticket**

d) A jar contains **4** red jelly beans, **3** yellow ones, and **3** green ones. You'll close your eyes and grab a jelly bean. Then you'll grab another one **without having put the first one back in the jar**.

A = The event that the **first** jelly bean is yellow

B = The event that the **second** jelly bean is also yellow

e) For the same jar of jelly beans, you'll close your eyes and grab a jelly bean. Then, **after putting it back in the jar**, you'll grab another one.

A = The event that the **first** jelly bean is yellow
B = The event that the **second** jelly bean is also yellow

The Multiplication Rule for Independent Events (4.6)

The Multiplication Rule for Independent Events

- The ***Multiplication Rule for Independent Events*** tells us how to find the probability of **two** independent events **both** happening:

Multiplication Rule for Independent Events: If A and B are two **independent** events, then

$$P(A \,\&\, B) \;=\; P(A) \times P(B)$$

Exercise

Consider tossing a coin twice. Find the **probability** that the coin will land heads on **both** tosses: *H* & *H*

Exercise

Roll pair of fair dice. Find the **probability** that **both** dice will land showing a six: ⚅ & ⚅

Exercise

A jar contains **4** red jelly beans, **3** yellow ones, and **3** green ones. You'll close your eyes and grab a jelly bean. Then **after putting it back in the jar**, you'll grab another one.

Exercise

A jar contains **4** red jelly beans, **3** yellow ones, and **3** green ones. You'll close your eyes and grab a jelly bean. Then **after putting it back in the jar**, you'll grab another one.

a) Find the **probability** that **both** jelly beans will be **yellow.**

Exercise

A jar contains **4** red jelly beans, **3** yellow ones, and **3** green ones. You'll close your eyes and grab a jelly bean. Then **after putting it back in the jar**, you'll grab another one.

a) Find the **probability** that **both** jelly beans will be **yellow**.

b) Find the **probability** that the **first** one will be **yellow** and the **second** one will be **red**.

- **Multiplication Rule for *More Than Two* for Independent Events**: The Multiplication Rule can be used with **any number of events.** For example, if A, B, and C are *three* (independent) events, then

$$P(A \,\&\, B \,\&\, C) \;=\; P(A) \times P(B) \times P(C)$$

Exercise

a) If we toss a coin **three** times, what's the **probability** that it will land heads on **all three** tosses: H & H & H.

Exercise

a) If we toss a coin **three** times, what's the **probability** that it will land heads on **all three** tosses: H & H & H.

b) If we toss it **eight** times, what's the **probability** that it will land heads on **all eight** tosses: H & H & H & H & H & H & H & H.

Exercise

a) If we roll three dice, what's the **probability** that **all three** will land on six: ⚅ & ⚅ & ⚅.

Exercise

a) If we roll three dice, what's the **probability** that **all three** will land on six: ⚅ & ⚅ & ⚅.

b) If we roll five dice, what's the probability that **all five** will land on six: ⚅ & ⚅ & ⚅ & ⚅ & ⚅.

Exercise

A jar contains **4** red jelly beans, **3** yellow ones, and **3** green ones. You'll blindly grab **three** jelly beans, one at a time, **each time replacing the jelly bean in the jar** before grabbing the next one.

Exercise

A jar contains **4** red jelly beans, **3** yellow ones, and **3** green ones. You'll blindly grab **three** jelly beans, one at a time, **each time replacing the jelly bean in the jar** before grabbing the next one.

a) Find the **probability** that **all three** jelly beans will be **yellow.**

Exercise

A jar contains **4** red jelly beans, **3** yellow ones, and **3** green ones. You'll blindly grab **three** jelly beans, one at a time, **each time replacing the jelly bean in the jar** before grabbing the next one.

a) Find the **probability** that **all three** jelly beans will be **yellow.**

b) Find the **probability** that the **first** one will be **yellow**, the **second** one **red**, and the **third** one **green.**