Introduction to Statistics

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Topics

Objectives

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- Compute and interpret a CI for a population proportion.
- Determine the sample size required for the margin of error in a CI for a population proportion to be no bigger than some specified value.

Confidence Interval for a Population Proportion P (12.1)

The Sample Proportion $\hat{P}$ and the Population Proportion P

- The mean $\bar{x}$ (or μ) is appropriate for summarizing a ***quantitative*** (numerical) variable.

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- The mean $\bar{x}$ (or μ) is appropriate for summarizing a ***quantitative*** (numerical) variable.
- For a ***qualitative*** (categorical) variable, the appropriate summary is the ***proportion*** of individuals in a category.

- The ***population proportion*** (a parameter) and ***sample proportion*** (a statistic) are defined on the next two slides.

Population Proportion: Consider a **population** in which each individual is classified according to a **qualitative** variable having **two categories**, **"success"** and **"failure"**, say.

The ***population proportion***, denoted p, is defined as:

$$p = \frac{\text{Number of "successes" in the population}}{N}$$

where N is the population size.

Sample Proportion: Now consider a **sample** of size n from the population just described.

The ***sample proportion***, denoted $\hat{p}$, is defined as:

$$\hat{p} = \frac{\text{Number of "successes" in the sample}}{n}$$

- The *sample proportion* $\hat{p}$ is used to **estimate** the *population proportion* p.

Example

Suppose a random **sample** of $n = 10$ students from a college was asked whether or not they smoke cigarettes (Yes or No), and that the resulting data are

Yes No Yes No Yes No No Yes No No

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Suppose a random **sample** of $n = 10$ students from a college was asked whether or not they smoke cigarettes (Yes or No), and that the resulting data are

Yes No Yes No Yes No No Yes No No

The **sample proportion** of smokers is

$$\hat{p} = \frac{4}{10} = \mathbf{0.4},$$

and so we'd **estimate** that the **true (unknown) proportion** p that smokes in the **population** is **0.4**, or **40%.**

- When the **sample proportion** $\hat{p}$ is to estimate a **population proportion** p, the **sampling error** is:

Sampling Error of the Sample Proportion:

$$\text{Sampling Error} = \hat{p} - p$$

The Sampling Distribution of the Sample Proportion $\hat{P}$

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- The **sampling distribution** of $\hat{p}$ can be used to gauge how large the **sampling error** of $\hat{p}$ might be when estimating p.
- In the slides ahead, we'll see that:

  As long as the **sample size n is large**, the **sampling distribution of** $\hat{p}$ will be **approximately normal**.

Normality of $\hat{P}$ When n is Large

Normality of $\hat{P}$: If we take a **sample** of size n from a **population** of **"successes"** and **"failures"** whose proportion of "successes" is p, then as long as the **sample size** n is ***large***:

The $\hat{p}$ **distribution** will be (at least approximately) ***normal*** with mean $\mu_{\hat{p}}$ and standard deviation $\sigma_{\hat{p}}$, where

$$\mu_{\hat{p}} = p \qquad \text{and} \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.$$

- **Interpretation of $\mu_{\hat{p}}$ and $\sigma_{\hat{p}}$:**
  - $\mu_{\hat{p}}$ is the value that $\hat{p}$ takes, **on average**. Thus, because $\mu_{\hat{p}} = p$, **on average** the **sample proportion** equals the **population proportion**.
  - $\sigma_{\hat{p}}$ represents a **typical deviation** of $\hat{p}$ away from p, i.e. a typical **sampling error**. Thus, because $\sigma_{\hat{p}} = \sqrt{p(1-p)/n}$, the size of a **typical sampling error** is $\sqrt{p(1-p)/n}$.

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- $\sqrt{p(1-p)/n}$ is often called the ***standard error*** of $\hat{p}$.

- The **standard error** of $\hat{p}$ **will be small** if either:

  1. The population proportion p **is close to 0 or 1** (i.e. the population is fairly ***homogeneous***).
  2. The sample size n **is large**.

  Under either of these conditions, $\hat{p}$ will be a **precise estimator** of p.

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First, though, we'll look at an example.

Exercise

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> *"A majority of Americans oppose legalizing the sale of human organs for transplant purposes."*

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The reported **margin of error** is ± 4.3 percentage points.

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e) Based on the **CI** of part *c*, is the conclusion stated in the first quote above justified?

Exercise

The Marist Poll described in the last exercise also reported the following result:

> *"In assessing the moral dimension of this debate,* **49%** *of U.S. residents believe it is wrong for someone to sell their organs, such as a kidney, to a transplant patient who can afford to pay the price."*

a) For this part of the survey, what is the **(unknown) population parameter** of interest?

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The Marist Poll described in the last exercise also reported the following result:

> *"In assessing the moral dimension of this debate,* **49%** *of U.S. residents believe it is wrong for someone to sell their organs, such as a kidney, to a transplant patient who can afford to pay the price."*

a) For this part of the survey, what is the **(unknown) population parameter** of interest?

b) What's the value of the **sample statistic** being used to **estimate** the **(unknown) parameter**?

c) Using the **estimate** and **margin of error** (± 4.3), determine the **CI** for p.

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d) Based on the **CI**, would it be reasonable to conclude that **fewer than half** of all Americans believe it's wrong for someone to sell their organs?

- To see how the **margin of error** is determined, recall that $\hat{p}$ follows a **normal** distribution with **mean** and **standard error**

$$\mu_{\hat{p}} = p \qquad \text{and} \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.$$

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... and therefore will lie between

$$-z_{0.025} = -1.96 \qquad \text{and} \qquad z_{0.025} = 1.96$$

95% of the time when we take a sample of size n.

- In other words,

$$-1.96 \leq \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \leq 1.96$$

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$$-1.96\sqrt{\frac{p(1-p)}{n}} \leq \hat{p} - p \leq 1.96\sqrt{\frac{p(1-p)}{n}}$$

which says the ***sampling error*** **won't be bigger than** $1.96\sqrt{p(1-p)/n}$ **95% of the time.**

- (cont'd)

  Subtracting $\hat{p}$ from all three terms gives

$$-\hat{p} - 1.96\sqrt{\frac{p(1-p)}{n}} \leq -p \leq -\hat{p} + 1.96\sqrt{\frac{p(1-p)}{n}}$$

- (cont'd)

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  Multiplying each of the three terms above by -1 (which changes the direction of the inequalities) gives

$$\hat{p} + 1.96\sqrt{\frac{p(1-p)}{n}} \geq p \geq \hat{p} - 1.96\sqrt{\frac{p(1-p)}{n}}.$$

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  Finally, reordering the terms, we get that **95% of the time**,

$$\hat{p} - 1.96\sqrt{\frac{p(1-p)}{n}} \leq p \leq \hat{p} + 1.96\sqrt{\frac{p(1-p)}{n}}.$$

- (cont'd)

  Thus, we can be **95% confident that p will be between**

$$\hat{p} - 1.96\sqrt{\frac{p(1-p)}{n}} \qquad \text{and} \qquad \hat{p} + 1.96\sqrt{\frac{p(1-p)}{n}}$$

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  Unfortunately, we ***can't*** use

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  as the margin of error in our CI because it depends on the **unknown** population proportion p.

  Instead, we plug in the **estimate $\hat{p}$** of p.

95% One-Proportion z CI for P: A 95% confidence interval for the **unknown** population **proportion** of "successes" p is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

and the **margin of error** is

$$\text{Margin of Error} = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

(These are valid when the sample is from a population of "**successes**" and "**failures**" and the sample size n is **large**.)

- For **other levels of confidence**, we replace 1.96 by the appropriate z ***critical value***:

Commonly Used Z Critical Values:

$$z_{0.05} = 1.645 \quad \text{for a 90\% level of confidence}$$
$$z_{0.025} = 1.96 \quad \text{for a 95\% level of confidence}$$
$$z_{0.005} = 2.58 \quad \text{for a 99\% level of confidence}$$

These $z_{\alpha/2}$ values are obtained from Table II.

One-Proportion z CI for p: The ***one-proportion z confidence interval*** for the **unknown** population **proportion** of "successes" p is

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

with ***margin of error***

$$\text{Margin of Error} = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where $z_{\alpha/2}$ is a z **critical value** and α is either **0.10**, **0.05**, or **0.01**, depending on the level of confidence (see the next slide).

$\alpha = \mathbf{0.10}$ for a **90%** level of confidence (so $1 - \alpha = \mathbf{0.90}$, $\alpha/2 = \mathbf{0.05}$, and $z_{\mathbf{0.05}} = \mathbf{1.645}$)

$\alpha = \mathbf{0.05}$ for a **95%** level of confidence (so $1 - \alpha = \mathbf{0.95}$, $\alpha/2 = \mathbf{0.025}$, and $z_{\mathbf{0.025}} = \mathbf{1.96}$)

$\alpha = \mathbf{0.01}$ for a **99%** level of confidence (so $1 - \alpha = \mathbf{0.99}$, $\alpha/2 = \mathbf{0.005}$, and $z_{\mathbf{0.005}} = \mathbf{2.58}$)

(The CI and margin of error are valid when the sample is from a population of "**successes**" and "**failures**" and the sample size n is **large**.)

Example

A June 2-7, 2015 Gallup poll of **1,527** adults, aged 18 and older, living in the U.S. found that **1,390** (or **91%**) of those surveyed would vote for a Hispanic for president.

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and this is the **estimate** of p, the true (unknown) population proportion of U.S. residents that would vote for a Hispanic.

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This gives a range plausible values for p, and we can be **95% confident** that p is in the interval somewhere.

For example, based on the **CI**, it's **not plausible** that p is as small as **0.85** because this value isn't in the interval.

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More precisely, it tells us that the **sampling error** is **not likely** to be **larger** than **0.01**.

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$$= 0.91 \pm 0.02$$

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Note that the **99%** CI is **wider** than the **95%** CI.

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a) **Compute** and **interpret** a **95% CI** for the **true (unknown) nationwide proportion** p that would vote for an atheist.

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a) **Compute** and **interpret** a **95% CI** for the **true (unknown) nationwide proportion** p that would vote for an atheist.

b) **Is it plausible**, based on the CI, that that p is **as large as 0.65**? Explain.

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b) **Is it plausible**, based on the CI, that that p is **as large as 0.65**? Explain.

c) How big is the **margin of error** in the estimate, $\hat{p} = 0.58$, of p? How do you **interpret** this value?

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c) How big is the **margin of error** in the estimate, $\hat{p} = 0.58$, of p? How do you **interpret** this value?

d) If a **99% CI** was computed instead, would the interval be **wider** or **narrower** than the **95% CI**?

(Optional Section) Determining the Required Sample Size for Estimating p (12.1)

- Recall that the **margin of error** in the **CI** for a population proportion p is

$$\text{Margin of Error} \quad = \quad z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\,.$$

(Optional Section) Determining the Required Sample Size for Estimating p (12.1)

- Recall that the **margin of error** in the **CI** for a population proportion p is

$$\text{Margin of Error} \quad = \quad z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\,.$$

We can make the **margin of error as small as we want** by using a **large enough** n.

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  But the right side depends on the value of the $\hat{p}$, which isn't known yet (because the sample hasn't been taken).

  Instead, we plug in a ***guess*** $\hat{p}_g$ for $\hat{p}$, which gives the ***required sample size*** (see the next slide).

Sample Size for Estimating μ: The sample size required for the margin of error in a CI for p to be no bigger than some value E is

$$n \geq \hat{p}_g(1-\hat{p}_g)\left(\frac{z_{\alpha/2}}{E}\right)^2,$$

which we **round up** to the nearest integer.

Above, $\hat{p}_g$ is a guess for the value of $\hat{p}$, either:

1. $\hat{p}_g$ is a guessed value for $\hat{p}$ (e.g. based on prior studies)

   or

2. $\hat{p}_g = 0.5$ is used as the guess for $\hat{p}$.

- The second option (using $\hat{p}_g = 0.5$) is **conservative** (i.e. **safe**) in that the resulting n will be ***as large or larger*** than what's actually needed, and so the resulting **margin of error** is **guaranteed** to be ***as small or smaller*** than E.

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If we want the **margin of error** in a **95% CI** for p to be **0.03** (i.e. **3 percentage points**), **how big should the sample size be**?

Using the guess $\hat{p}_g = \mathbf{0.5}$ in the sample size calculation, we'd need a **sample size**

$$\begin{aligned} n &\geq \hat{p}_g(1-\hat{p}_g)\left(\frac{z_{0.025}}{E}\right)^2 \\ &= 0.5 \times (1-0,5)\left(\frac{1.96}{0.03}\right)^2 \\ &= \mathbf{1067.1}, \end{aligned}$$

which we **round up** to $n = \mathbf{1,068}$.

Exercise

Suppose instead that in our poll of Colorado voters, we only need the margin of error to be **0.04** (i.e. **4 percentage points**).

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Use the guess $\hat{p}_g = 0.5$ in the sample size calculation.