Introduction to Statistics

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Topics

Objectives

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- Carry out a one-mean t test for a population mean μ when the population standard deviation σ is unknown.

Hypothesis Test for μ when σ is Unknown (9.5)

Introduction to the One-Mean t Test

- Usually the population standard deviation σ ***isn't*** **known**, so we **can't** use the *one-mean z test* for μ.

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  Instead, we **estimate** σ by the sample standard deviation s, and then use the ***one-mean t test*** described ahead.

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Introduction to the One-Mean t Test

- Usually the population standard deviation σ ***isn't*** **known**, so we **can't** use the *one-mean z test* for μ.

  Instead, we **estimate** σ by the sample standard deviation s, and then use the ***one-mean t test*** described ahead.

  The test is used when we have a **random sample** from a **population**.

  We'll use the sample to decide if the population mean μ is different from some **claimed value** μ_0.

- The **null hypothesis** is:

Null Hypothesis:

$$H_0 : \mu = \mu_0$$

- The **alternative hypothesis** will depend on what we're trying to **substantiate**:

Alternative Hypothesis: The alternative hypothesis will be one of

1. $H_a : \mu > \mu_0$ (**one-sided, upper-tailed test**)
2. $H_a : \mu < \mu_0$ (**one-sided, lower-tailed test**)
3. $H_a : \mu \neq \mu_0$ (**two-sided, two-tailed test**)

depending on what we're trying to substantiate in our study.

- The **test statistic** for the ***one-mean t test for μ*** is

One-Mean T Test Statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

When

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is true, the **sampling distribution** of the test statistic t is a t distribution with $n-1$ df.

(The *one-mean* t *test* is valid if either the sample is from a **normal** population or the sample size n is **large** ($n \geq 30$).)

- t measures (approximately) how many **standard errors** $\bar{x}$ is **away from** the claimed value μ_0.

1. Values of t **close to zero** provide almost ***no* evidence against** the **null hypothesis** $H_0 : \mu = \mu_0$.

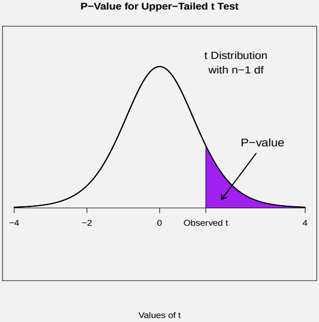
2. **Positive** values of t provide **evidence against the null hypothesis in favor of** $H_a : \mu > \mu_0$.

3. **Negative** values of t provide **evidence against the null hypothesis in favor of** $H_a : \mu < \mu_0$.

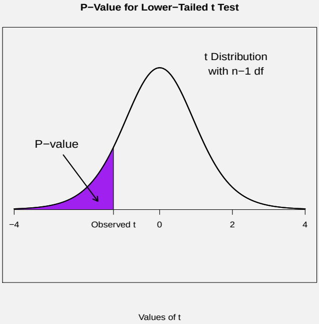
4. **Positive *and* negative** values of t provide **evidence against the null hypothesis in favor of** $H_a : \mu \neq \mu_0$.

- The ***p-value*** is the probability that just by chance (under H_0) we'd get a test statistic value as far from zero, in the direction predicted by H_a, as the observed value.

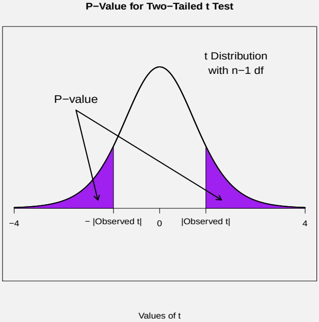
1. **P-value** = Area to the **right** of the observed t if the alternative hypothesis is $H_a : \mu > \mu_0$.



2. **P-value** = Area to the **left** of the observed t if the alternative hypothesis is $H_a : \mu < \mu_0$.



3. **P-value** = Area to the **left** of $-|t|$ **and right** of $|t|$ if the alternative hypothesis is $H_a : \mu \neq \mu_0$.



- As always, after choosing a **level of significance** α, we **reject** $\boldsymbol{H_0}$ if **p-value** $< \alpha$, otherwise fail reject H_0.

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The machine will need to be adjusted if the boxes are systematically being **under-filled** or **over-filled**.

To decide if the boxes are being **under-filled or overfilled**, the engineer will test the **hypotheses**

$$H_0: \mu = 16$$
$$H_a: \mu \neq 16$$

where μ is the true (unknown) population mean weight.

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where μ is the true (unknown) population mean weight.

A random sample of **ten** boxes gives

$$\bar{x} = \mathbf{16.6} \qquad \text{and} \qquad s = \mathbf{0.9}.$$

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Thus the **sample mean** weight, $\bar{x} = \mathbf{16.6}$, is about **2.11 standard errors above 16** ounces.

The **p-value** is the **probability** that we'd get a t value this far away from zero (in either direction) by chance **if** the **population mean** μ was **16**.

From the **two tail** areas of the t **distribution** with $n - 1 = 9$ **df**, to the **right** of **2.11** and **left** of **-2.11**,

$$\textbf{p-value} = 2 \times 0.033 = \mathbf{0.066}.$$

(The value 0.033 was obtained from the **table** showing **areas to the right of** t.)

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Thus we'd get a result like the one we got **6.6%** of the time **even if** the **population mean** μ was **16** ounces.

Using a **level of significance** $\alpha = \mathbf{0.05}$, the **decision rule** is

Reject H_0 if p-value < 0.05.
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The result that the engineer got (by taking a random sample) can be explained by chance variation (sampling error).

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The data are the ***changes*** in blood pressure for the 10 men:

-7   4   -18   -17   3   5   -1   -10   -11   2

A **negative** value means the **blood pressure decreased.**

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Hint: You should get **-1.81**.

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c) Determine the **p-value**.

Hint: You should get **0.053**.

d) State the **conclusion** using a **level of significance** $\boldsymbol{\alpha = 0.05}$, in which case the **decision criterion** is

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e) **Interpret** the result: Is there **statistically significant** evidence that calcium **lowers blood pressure**?

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e) **Interpret** the result: Is there **statistically significant** evidence that calcium **lowers blood pressure**?

f) If instead we had used a **significance level** $\boldsymbol{\alpha = 0.10}$, would the conclusion have been different? Explain

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Exercise

In the U.S., the mean number of newly decayed teeth an individual gets per year is **0.30**.

To investigate the **claim** that **sugar increases the decay rate of teeth**, **25** adults were examined and then given a sugar solution to supplement all their meals.

After one year, the mean and standard deviation of the number of newly decayed teeth for the group were

$$\bar{x} = \mathbf{0.60} \qquad \text{and} \qquad s = \mathbf{0.45}.$$

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Hint: You should get **0.002**.

Data Snooping: Don't Do It!

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  If you don't have a particular direction in mind for H_a *before* examining the data, *use the two-tailed test*.

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- The next example shows how **data snooping** can lead an **artificially small p-value** and in turn to **mistakenly rejecting H_0** when you shouldn't have.

Example

Suppose in the cereal-box weight study from the earlier example that the engineer "**cheats**" (**data snoops**) and decides, ***after*** noticing that $\bar{x} = \mathbf{16.6}$ is ***greater*** than **16**, to do an **upper-tailed test** of

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What would the **p-value** be?

Answer: **0.033** (just the **upper** tail area).

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Answer: **Yes.** In this case, the **null hypothesis** would (mistakenly) be **rejected.**