Introduction to Statistics

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Notes

Topics

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Objectives

Objectives:

- Compute and interpret the R-squared associated with a linear regression analysis.

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Assessing the Fit of a Regression Line Using R Squared (14.3)

Introduction

- The square of the correlation, r^2, is called the ***coefficient of determination***, but is more often referred to as **"R squared"**.
- r^2 is used to **measure how well** the **regression line fits** the **data** in a scatterplot:
  - Values of r^2 **close** to **1.0** mean the regression line **fits** the data quite **well**.
  - Values **close** to **0** mean it ***doesn't* fit** very well at all.

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- More formally, r^2 represents ***the proportion of variation in y that can be explained by variation in x.***

  What does this mean? The value of y will **vary** from one individual to the next.

  We can attribute this y **variation** to **two sources**:

  1. **Variation** in y **due** to **variation in** x.

     This is what leads to an overall upward or downward **sloping pattern** in the scatterplot.

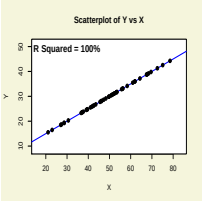
  2. **Variation** in y **due** to **variation in all *other* factors (*besides* x)**.

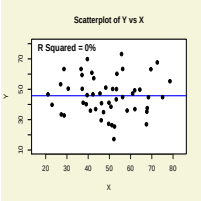
     This is what leads to residual **errors** in the scatterplot.

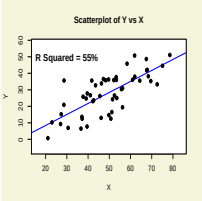
- Now consider the two extremes:
  - If **all** (**100%**) of the variation in y was due to x, ...

    ... the points in the scatterplot would lie **exactly** on a **straight line**, and r^2 would equal **1.0**.
  - If **none** (**0%**) of the variation in y was due to x (i.e. *all* of it was due to other factors *besides* x), ...

    ... the points in the scatterplot would form a **round "blob"**, and r^2 would equal **0.0**.
- Most data sets fall somewhere between these two extremes.





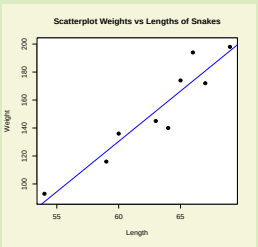


Exercise

The data and scatterplot (with fitted regression line) for the **lengths** (x) and **weights** (y) of snakes are below.

Lengths and Weights of Female Snakes

Snake	Length (cm)	Weight (g)
1	60	136
2	69	198
3	66	194
4	64	140
5	54	93
6	67	172
7	59	116
8	65	174
9	63	145



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Statistical software gives the **equation** of the **regression line**:

$$\hat{y} = -300.97 + 7.19x$$

The **correlation** between **lengths** and **weights** is

$$\boldsymbol{r = 0.94}.$$

a) Note: the **weight** of a snake isn't solely determined by its **length** (otherwise the points in the scatterplot would lie exactly on the line).

List a few of the other factors (*besides* length) that determine how much a snake weighs.

b) Calculate $\boldsymbol{r^2}$, the **coefficient of determination**.

c) Based $\boldsymbol{r^2}$, what **percentage** of the **variation** in snakes' **weights** that's attributable to **variation** in their **lengths**?

What **percentage** is due to **all *other* factors *besides* length**?

(Optional Section) An Explanation for Why r^2 Measures the Proportion of y Variation That's Due to x (14.3)

- To see why r^2 is interpreted as the *proportion of variation in y that can be explained by x*:
  - First consider the ***total sum of squares of y***, **SST**, defined to be

$$\text{SST} = \sum_{i=1}^{n}(y_i - \bar{y})^2,$$

the sum of squared deviations of the observed y_i's away from their mean $\bar{y}$.

SST is a measure of the **total variation** in the **observed y values**.

- (cont'd)
  - Second, consider the ***error sum of squares***, **SSE**, defined to be

$$\text{SSE} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2,$$

the **"sum of squared errors"** (deviations of the observed y_i's away from their predicted values $\hat{y}_i$).

SSE is a measure of the **variation** in the y **values** that's due to **all other factors *besides* x**.

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- (cont'd)
  - The ratio **SSE**/**SST** has the following interpretation:

$$\frac{\text{SSE}}{\text{SST}} = \frac{\text{Variation in } y \text{ that's } \textit{not} \text{ due to } x}{\text{Total variation in } y}$$

$$= \text{Proportion of the total variation in } y \text{ that's } \textit{not} \text{ due to } x.$$

Notes

- (cont'd)
  - It can be shown that

$$\boldsymbol{r^2 = 1 - \frac{\text{SSE}}{\text{SST}}}.$$

The right side has the following interpretation:

$$1 - \frac{\text{SSE}}{\text{SST}} = 1 - \text{The proportion of the total variation in } y \text{ that's } \textit{not} \text{ due to } x$$

$$= \textbf{The proportion that } \textbf{\textit{is}} \textbf{ due to } \boldsymbol{x}.$$

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